

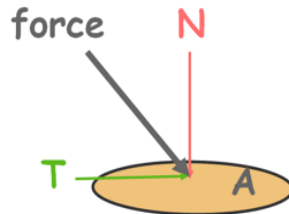
UNIT -III

STRESSES IN SOILS

Total stress at a layer present at a certain depth below ground level is the **total weight of the soil** present above that layer **per unit surface area of the soil mass**. **Effective stress** is the **part of the total stress that is resisted by soil particles** by grain-to-grain interaction.

TOTAL STRESS CONCEPT

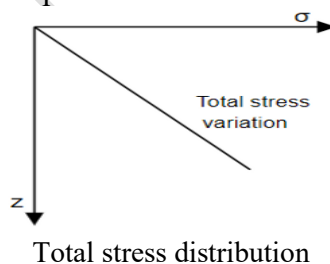
Total stress at a layer is the **total weight of the soil present above that layer**, including self-weight of water present in soil pores or present above the ground level, **per unit surface area of the soil**. Total stress also includes the stresses due to externally applied loads, if any. It expresses the internal forces that particles of a material exert on each other.



The stress is a vector quantity just like the force. So we can decompose it into normal stress (σ) and shear stress (τ). The stress developed on a surface because of the force component perpendicular to its area is called normal stress. N is the force component normal to the surface of area A . And the stress developed in the surface because of the force component tangential to its area is called shear stress.

$$\sigma = \frac{N}{A} \quad \tau = \frac{T}{A}$$

The pattern of stresses due to self-weight could be complicated except for one common case where the stresses are simple. One common case is when the **ground surface is horizontal** and the **nature of the soil does not vary significantly in the horizontal directions**. This is the most frequent case in any sedimentary deposit. Also, in this case, there can be no shear stresses upon vertical and horizontal planes within the soil mass.



Total Stress Formula

Total stress (σ_v) at a layer present at a depth of ' z ' from the ground surface is given by,

$$\sigma = \gamma * z,$$

provided **unit weight (γ)** of the soil is **constant**.

where, σ - total vertical stress (kN/m^2),

γ - the unit weight of soil (kN), z - depth under consideration (m).

PORE WATER PRESSURE CONCEPT

As we all know, the soil is a [three-phase system](#) i.e., it contains solid, porewater and air voids. In the case of saturated soils, the pressure exerted by the pore water to its surrounding solid mass is called pore water pressure. Pore water pressure is also called **Neutral stress**.

Neutral stress can be defined as the **stress carried** (exerted) **by the pore water** and it is the same in all directions when there is static equilibrium.

Pore water pressure(u) at a depth z can be calculated as the product of the unit weight of water and depth as given below.

$$u = \gamma_w * z,$$

where,

γ_w - the unit weight of the water (9.81 kN/m³),

z - depth at which pore water pressure is required.

- Pore water pressure can become negative in the case of [capillary water](#). In such a case, pore water pressure (u) is calculated as,

$$u = -\gamma_w * h,$$

where, h - the height of the capillary rise above the ground level.

🔧 EFFECTIVE STRESS CONCEPT

The total stress at any point inside the soil is resisted by the soil grains and also by water present in the pores or void spaces (in the case of saturated soils). This portion of total stress that is resisted by soil grains is called effective stress. Effective stress is in macroscopic scale i.e., force/total area, and not in terms of contact stress at the grain-to-grain contacts as these will be very high.

In simple terms, imagine total stress as downward stress that is being resisted by effective stress by virtue of soil grains and pore water pressure by virtue of pore water, both acting in the upward direction.

Effective Stress Formula

Effective stress cannot be directly calculated but it can be calculated as the difference between total stress(σ) and pore water pressure(u). It is represented using σ' .

$$\text{Effective stress, } \sigma' = \text{Total stress } (\sigma) - \text{Pore water pressure}(u)$$

• Example problem

Question: Determine total, neutral, and effective stress at a depth of 16m below the ground. The water table is 4m below the ground level. Take $\gamma_{\text{bulk}} = 16.5 \text{ kN/m}^3$, and $\gamma_{\text{sat}} = 19.4 \text{ kN/m}^3$.

Solution:

Take $z_1 = 4\text{m}$, and $z_2 = 12\text{m}$ (i.e., $16 - 4$)

Total stress at depth of 4m, $\sigma = \gamma_{\text{bulk}} * z_1$

$$\sigma = 16.5 * 4 = 66 \text{ kN/m}^2$$

Total stress at depth of 16m, $\sigma = \gamma_{\text{bulk}} * z_1 + \gamma_{\text{sat}} * z_2$

$$\sigma = 16.5 * 4 + 19.4 * 12 = 298.8 \text{ kN/m}^2$$

Neutral stress at depth of 16m, $u = \gamma_w * z_2$

$$u = 9.81 * 12 = 117.72 \text{ kN/m}^2$$

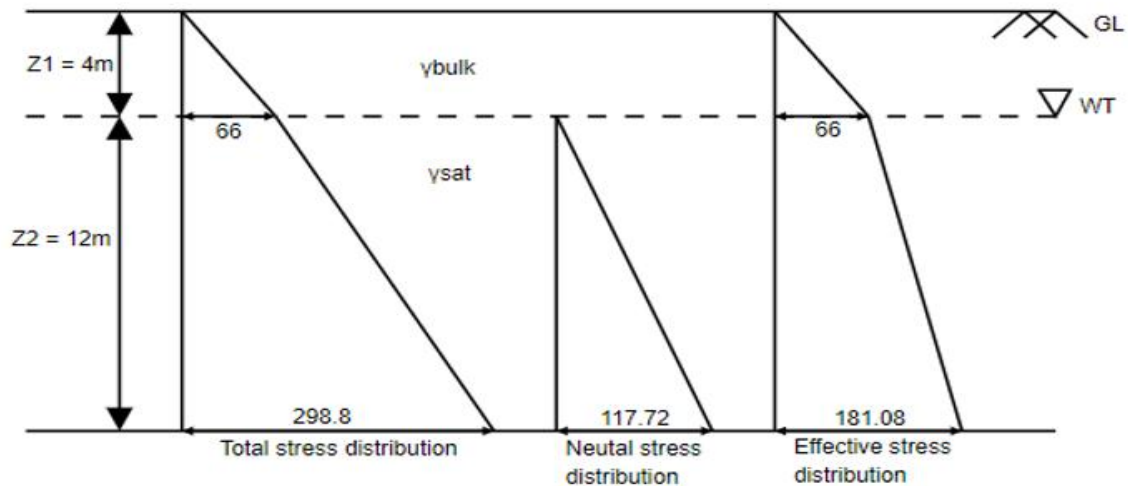
Effective stress at a depth of 4m, $\sigma' = \sigma - u$

$$\sigma' = 66 - 0 = 66 \text{ kN/m}^2$$

Effective stress at a depth of 16m, $\sigma' = \sigma - u$

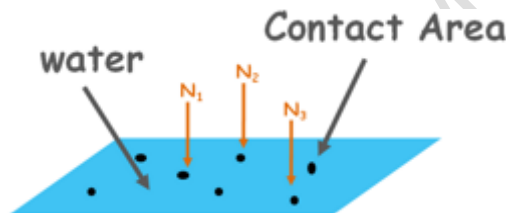
$$\sigma' = 298.8 - 117.72 = 181.08 \text{ kN/m}^2$$

Note: All units are in kN/m² in the below-given diagram.



Total, neutral and effective distribution

EFFECTIVE STRESS IS DIFFERENT AT DIFFERENT DEPTHS BELOW GROUND SURFACE



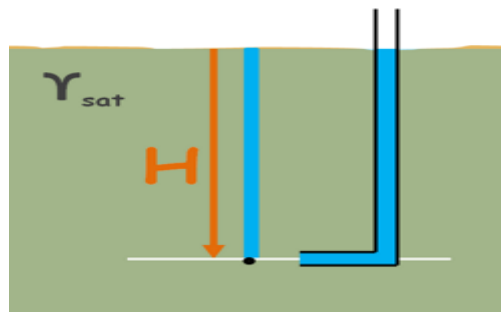
Let us consider a soil mass and try to find out the effective stress at some point below ground at depth capital H.



$$\sigma = \gamma H$$

In our case we have taken unit weight of the soil as saturated unit weight.

$$\sigma = \gamma_{\text{sat}} H$$



Now the pore water pressure at this point in the soil is the pressure due to the weight of water column above it of height H. So pore water pressure at this point is also the weight of water column divided by the area. So pore water pressure can be written, similar to the total normal stress, as this.

$$u = \gamma_w H$$

Here γ_w is unit weight of water. We will find this pore water pressure similar if we insert a tube or a standpipe at this point in the soil and observe the rise of water because water in the soil is also at atmospheric pressure and water level will be the piezometric level.

Therefore, the effective stress at this point in the soil can be given as this.

$$\sigma' = \sigma - u = \gamma_{\text{sat}}H - \gamma_w H$$

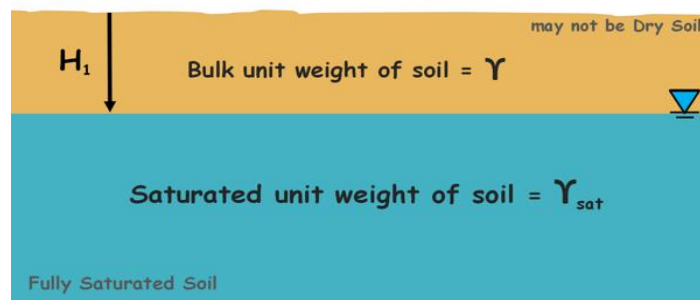
The increase in the effective stress causes the particles to pack more closely which results in decrease in the void ratio because of decrease in volume of voids and that leads to a decrease in compressibility and increase the shearing resistance of the soil.

In the calculation of effective stress we assume that the soil is isotropic that means the unit weight of the soil is constant throughout the soil layer. But in the actual field conditions soil structure is non homogeneous and not isotropic and contains various defects.

Soil particles in a soil mass experience weight of everything above them which includes weight of all the soil particles above them, weight of water and weight of structure if present, and because of that soil particles experience stress in between each other which we call total stress.

Effective stress is different at different depths below ground surface because total stress and pore water pressure changes at different depth.

If level of water table in the soil changes because of any external factors like rain or increased overburden pressure, the movement of water takes place inside the voids present in the soil mass. This state of movement of water inside the soil is called the hydro dynamic state or transient state of water. So let us consider this soil mass in which the position of the water table is H_1 depth below the ground surface.



Soil below the ground water table is fully saturated and has unit weight γ_{sat} , while soil above the water table may not be completely dry hence its unit weight is considered bulk unit weight γ .

So on a plane that lies within the saturated layer of the soil deposit total downward stress is equal to the stress because of soil and water above it. Depth of unsaturated soil layer is assumed H_1 and let the plane where we are calculating the stress is at depth H_2 from the water table.



Hence total stress on this plane because of this not-saturated soil mass is γH_1 and stress due to saturated soil mass is $\gamma_{\text{sat}} H_2$.

$$\sigma = \gamma H_1 + \gamma_{\text{sat}} H_2$$

Now the pore water pressure at this plane is because of water present above this plane up to the height of H_2 . So pore water pressure on this plane is
 $u = \gamma_w H_2$

Hence we can calculate the effective stress on this plane in the soil as, $\sigma' = \sigma - u$

By writing the corresponding values we get this: $\sigma' = (\gamma H_1 + \gamma_{\text{sat}} H_2) - (\gamma_w H_2)$

and by rearranging it we can also write it as this: $\sigma' = \gamma H_1 + (\gamma_{\text{sat}} - \gamma_w) H_2$

if we remember, the quantity in the bracket is called submerged unit weight. so we can write this expression as : $\sigma' = \gamma H_1 + \gamma' H_2$
 γ' is the submerged unit weight of soil.

✚ **IF THE WATER TABLE RISES TO THE GROUND**, the whole soil deposit becomes completely saturated.



Total stress at this plane is now caused by the whole saturated soil mass which we can write as saturated unit weight of soil multiplied by the total height of the soil mass.

$$\sigma = \gamma_{\text{sat}}(H_1 + H_2)$$

Pore water pressure here becomes unit weight of water multiplied by soil deposit height.

$$u = \gamma_w(H_1 + H_2)$$

Hence effective stress can be calculated as this. $\sigma' = \sigma - u$

$$\sigma' = \gamma_{\text{sat}}(H_1 + H_2) - \gamma_w(H_1 + H_2)$$

$$\sigma' = (\gamma_{\text{sat}} - \gamma_w)(H_1 + H_2)$$

We write this as submerged unit weight of soil, $\sigma' = \gamma'(H_1 + H_2)$

✚ **IF THE WATER TABLE FALLS BELOW OUR OBSERVATION PLANE** then pore water pressure becomes zero, as no water present in the voids to apply pressure.



Effective stress will be the total stress which is the bulk unit weight of the soil multiplied by height of the soil mass.

$$\sigma' = \sigma = \gamma(H_1 + H_2)$$

Again if we compare this result with the result of the initial partially saturated condition, we observe because of the water drawdown the effective stress has increased.

Now consider a different soil condition where soil mass consists of two different soil deposits which have different bulk unit weights γ_1 and γ_2 .



Let us calculate the values of effective stress at different sections of this soil deposit with different hydrostatic conditions. Let's begin by imagining the water table is above the soil mass and is of height H_0 . These soils become saturated and their bulk unit weights become γ_{sat1} and γ_{sat2} . Thickness of top layer is H_1 and bottom layer is H_2 .



On this plane the downward stress is caused by the weight of above material which here is water only. So, total stress on this plane is unit weight of water multiplied by the height of the water layer above this plane section.

$$\sigma = \gamma_w H_0$$

Now the pore water pressure on this plane caused by the water above is unit weight of water multiplied by the height of the water.

$$u = \gamma_w H_0$$

Hence effective stress on this plane is, $\sigma' = \sigma - u$

$$\sigma' = \gamma_w H_0 - \gamma_w H_0$$

so effective stress is zero., $\sigma' = 0$

Effective stress is the stress transferred at the contact of soil particles, but in this example above our plane of consideration there are no soil particles present. So no inter-particle force transfer, hence effective stress on this plane is zero.

Now move down a little and let us calculate the effective stress on a plane section within the top soil layer, which is H_1' depth below the actual soil surface.



Total stress on this plane section is caused by the above water layer and this soil mass. Stress caused by water layer is $\gamma_w H_0$ and stress caused by the saturated soil layer above the plane is $\gamma_{(sat)1} H_1'$.

$$\sigma = \gamma_w H_0 + \gamma_{(sat)1} H_1'$$

Water present in the soil voids will also cause to generate the vertical downward stress on the plane because of its weight, but in the calculation of total stress we have taken the saturated unit weight of the soil that includes the weight soil particles and weight of water as well.

Now pore water pressure at this plane is because of the whole water available above this plane. Which is γ_w multiplied by whole height of water surface.

$$u = \gamma_w(H_0 + H_1')$$

Hence effective stress can be calculated as, $\sigma' = \sigma - u$

$$\sigma' = (\gamma_w H_0 + \gamma_{(sat)1} H_1') - (\gamma_w H_0 + \gamma_w H_1')$$

$$\sigma' = (\gamma_{(sat)1} - \gamma_w) H_1'$$

it can be written as this: $\sigma' = \gamma' H_1'$

To calculate the effective stress at the plane, where the two soil layers meet each other, the expression for effective stress will remain the same and only the depth of our observation plane from the soil surface, which is the only variable here, changes to H_1 , which is now the depth of our new observation plane from the soil surface.

$$\sigma' = \gamma' H_1$$

 **Calculate the effective stress on a plane somewhere in the middle of bottom soil layer.** Let's say the depth of the observation plane in the bottom layer is H_2' .



total stress on this plane is stress due to the top water layer $\gamma_w H_0$, stress due to the saturated top soil layer $\gamma_{(sat)1} H_1$, and stress due to saturated bottom soil layer above plane $\gamma_{(sat)2} H_2'$.

$$\sigma = \gamma_w H_0 + \gamma_{(sat)1} H_1 + \gamma_{(sat)2} H_2'$$

pore water pressure on this plane is γ_w multiplied by total height of the water above the plane

$$u = \gamma_w(H_0 + H_1 + H_2')$$

So effective stress can be calculated as this

$$\sigma' = \sigma - u$$

$$\sigma' = (\gamma_w H_0 + \gamma_{(sat)1} H_1 + \gamma_{(sat)2} H_2') - (\gamma_w H_0 + \gamma_w H_1 + \gamma_w H_2')$$

$$= (\gamma_{(sat)1} - \gamma_w) H_1 - (\gamma_{(sat)2} - \gamma_w) H_2'$$

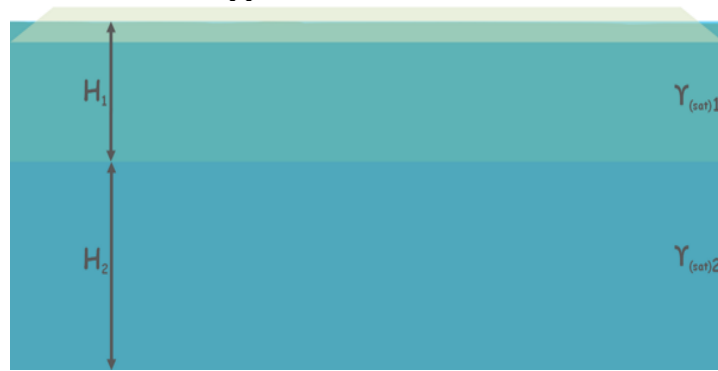
which can be written as $\sigma' = \gamma_1' H_1 - \gamma_2' H_2'$

If we need to calculate the effective stress at the bottom of the bottom soil layer, the expression for effective stress will again remain the same and only the depth of our observation plane H_2' , which is the only variable here, changes to the thickness of the soil layer H_2 .

$$\sigma' = \gamma_1' H_1 - \gamma_2' H_2$$

Hence with all our calculations we observed that change in the water level above the ground does have no effect on effective stress and it remains unchanged.

- the case when water level has dropped to the soil surface.



The plane section at the surface has nothing above it and total stress is zero and so is the pore water pressure. Consequently effective stress at this section is zero.

$$\sigma' = \sigma - u$$

$$\sigma' = 0 - 0 = 0$$

- Now let us calculate the effective stress on the plane where these soils meet each other. We can also take this plane anywhere in the top soil layer but result will be similar and only the depth of the observation will change.



Total stress on this plane due to saturated soil layer above, $\gamma_{sat1}H_1$.

$$\sigma = \gamma_{sat1}H_1$$

and pore water pressure is $\gamma_w H_1$

$$u = \gamma_w H_1$$

hence effective stress will be, $\sigma' = \sigma - u$

$$\sigma' = (\gamma_{sat1}H_1) - (\gamma_w H_1)$$

$$\sigma' = (\gamma_{sat1} - \gamma_w)H_1$$

and this can be written as this : $\sigma' = \gamma'_1 H_1$

Now on the plane section at the bottom of soil deposit



the total stress is due to saturated soil layers above the plane is

$$\sigma = \gamma_{sat1}H_1 + \gamma_{sat2}H_2$$

Pore water pressure at this plane is $u = \gamma_w H_1 + \gamma_w H_2$

Hence effective stress will be $\sigma' = \sigma - u$

$$\sigma' = (\gamma_{(sat)1}H_1 + \gamma_{(sat)2}H_2) - (\gamma_w H_1 + \gamma_w H_2)$$

$$\sigma' = (\gamma_{(sat)1} - \gamma_w)H_1 + (\gamma_{(sat)2} - \gamma_w)H_2$$

Which we write in submerged unit weight of soils as this. $\sigma' = \gamma_1' H_1 + \gamma_2' H_2$

Now water level in the soil has dropped and water table is now somewhere within the top layer of the soil mass. Again we will calculate the effective stress on these different sections.



At the top layer as we have already seen there will be no stress and effective stress is zero. Now at the level where soil deposit meets each other:



Total stress at this plane is the sum of stress due to the soil above this plane which is both saturated and not saturated. Stress due to not saturated part of soil is bulk unit weight of soil γ_1 multiplied by the height of unsaturated soil, say H_1' , and stress due to saturated part of the soil is $\gamma_{(sat)1}$ multiplied by saturated height of water above the plane, say H_1'' .

$$\sigma = \gamma_1 H_1' + \gamma_{(sat)1} H_1''$$

pore water pressure is γ_w and height of the water above plane, $u = \gamma_w H_1''$

So effective stress is $\sigma' = \sigma - u$

$$\sigma' = (\gamma_1 H_1' + \gamma_{(sat)1} H_1'') - (\gamma_w H_1'')$$

$$\sigma' = \gamma_1 H_1' + (\gamma_{(sat)1} - \gamma_w) H_1''$$

and this can be written in submerged unit weight as this: $\sigma' = \gamma_1 H_1' + \gamma_1' H_1''$

Now if we calculate the effective stress at the bottom of the second layer.



Total stress at this plane is unit weight of unsaturated soil 1 $\gamma_1 H_1$ plus stress due to saturated soil 1 $\gamma_{(sat)1} H_1''$ plus stress due to the whole saturated bottom soil layer $\gamma_{(sat)2} H_2$

$$\sigma = \gamma_1 H_1 + \gamma_{(sat)1} H_1'' + \gamma_{(sat)2} H_2$$

Pore water pressure will be because of total water layer above

$$u = \gamma_w H_1'' + \gamma_w H_2$$

Hence effective stress can be calculated as this

$$\sigma' = \sigma - u$$

$$\sigma' = (\gamma_1 H_1 + \gamma_{(sat)1} H_1'' + \gamma_{(sat)2} H_2) - (\gamma_w H_1'' + \gamma_w H_2)$$

$$\sigma' = \gamma_1 H_1 + (\gamma_{(sat)1} - \gamma_w) H_1'' + (\gamma_{(sat)2} - \gamma_w) H_2$$

Which is also written as this $\sigma' = (\gamma_1 H_1 + \gamma_1' H_1'' + \gamma_2' H_2)$

Now we assume water level has dropped even further and water table is now somewhere within the bottom layer of the soil mass.



Effective stress on the surface of the soil deposit will be zero

Now the stress at the plane section where soil deposits meet will only because unsaturated soil layer above.



So total stress, at this plane, is bulk unit weight of soil, γ_1 , multiplied by thickness of layer H_1 .

$$\sigma = \gamma_1 H_1$$

No water is present above this plane so pore water pressure is zero here, $u = 0$

Hence effective stress will be equal to the total stress.

$$\sigma' = \gamma_1 H_1 - 0 = \gamma_1 H_1$$

Finally we calculate the effective stress at the bottom most plane section.



Total stress on the bottom most plane is stress due to top unsaturated soil layer $\gamma_1 H_1$ plus stress due to unsaturated part of the bottom soil deposit of thickness say H_2' plus stress due to saturated part of the bottom soil deposit $\gamma_{(sat)2} H_2''$.

$$\sigma = \gamma_1 H_1 + \gamma_2 H_2' + \gamma_{(sat)2} H_2''$$

here pore water pressure will be, $u = \gamma_w H_2''$

So effective stress will be $\sigma' = \sigma - u$

$$\sigma' = (\gamma_1 H_1 + \gamma_2 H_2' + \gamma_{(sat)2} H_2'') - (\gamma_w H_2'')$$

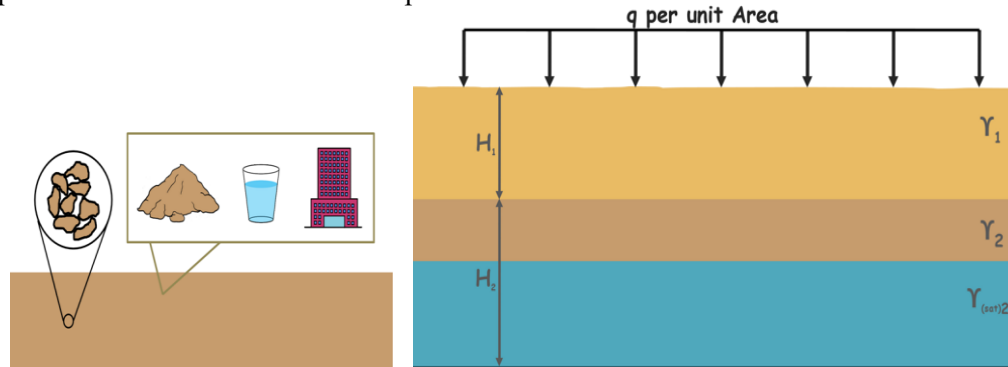
$$\sigma' = \gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{\text{sat}2} - \gamma_w) H_2''$$

part of the equation can also be written as submerged unit weight of bottom soil layer

$$\sigma' = \gamma_1 H_1 + \gamma_2 H_2' + \gamma_2' H_2''$$

EFFECTIVE STRESS DUE TO SURCHARGE

Soil particles in a soil mass experience weight of everything above them which includes weight of all the soil particles above them, weight of water and weight of structure if present, and because of that they experience stress in between the soil particles which we call total stress.



Consider a case where a soil mass, which consists of two soil deposits of bulk unit weight γ_1 and γ_2 , of thickness H_1 and H_2 respectively, is subjected to a surcharge. Surcharge is the additional load on the soil which may be created due to any overlying structure or any moving object.

Let us assume that soil is subjected to a surcharge of intensity q per unit area. Water table lies somewhere inside the bottom soil deposit. Because of the water table soil becomes completely saturated and unit weight of soil becomes $\gamma_{\text{sat}2}$.

Notice that surcharge load is given as force per unit area, which is equivalent to pressure or stress. So it can be directly used in the equations to provide stress at any depth.

Let's begin with the soil surface. Total stress on the surface of the soil is because of the surcharge only. So total stress is equal to the surcharge q .

$$\sigma = q$$

No water present above this plane, hence pore water pressure is zero., $u = 0$

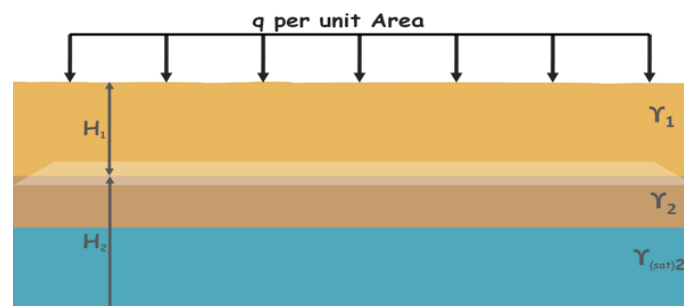
So effective stress which is total stress minus pore water pressure is q .

$$\sigma' = \sigma - u$$

$$\sigma' = q - 0$$

$$\sigma' = q$$

Now move a little downward and calculate effective stress at the plane where the two soils meet each other.



We can also take this plane anywhere in the top soil layer but result will be similar and only the depth of the observation will change. Total stress at this plane is surcharge load q plus stress due to weight of the soil layer above $\gamma_1 H_1$.

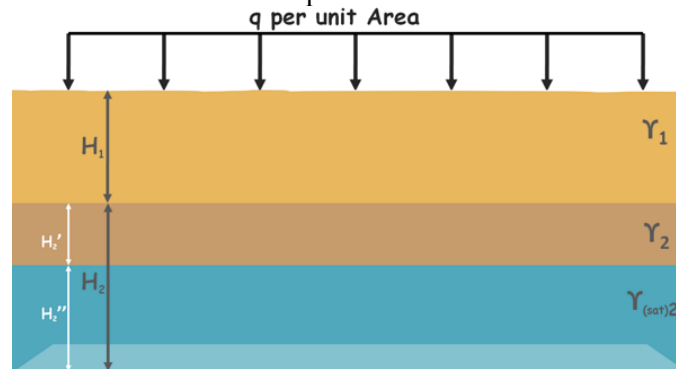
$$\sigma = q + \gamma_1 H_1$$

No water is present above this plane so pore water pressure is zero, $u = 0$

So effective stress is equal to the total stress. $\sigma' = \sigma - u$

$$\sigma' = \sigma = q + \gamma_1 H_1$$

Now let us calculate the effective stress on the plane at the bottom of the bottom soil deposit.



Total stress at this plane is surcharge stress q plus stress because of top not saturated soil layer $\gamma_1 H_1$ plus stress because of the unsaturated part of the lower soil deposit of thickness say H_2' , $\gamma_2 H_2'$ plus stress due to saturated soil of thickness say H_2'' , $\gamma_{(sat)2} H_2''$.

$$\sigma = q + \gamma_1 H_1 + \gamma_2 H_2' + \gamma_{(sat)2} H_2''$$

Pore water pressure at this plane is because of this water present above it

$$u = \gamma_w H_2''$$

Hence effective stress is total stress minus pore water pressure.

$$\sigma' = \sigma - u$$

$$\sigma' = (q + \gamma_1 H_1 + \gamma_2 H_2' + \gamma_{(sat)2} H_2'') - (\gamma_w H_2'')$$

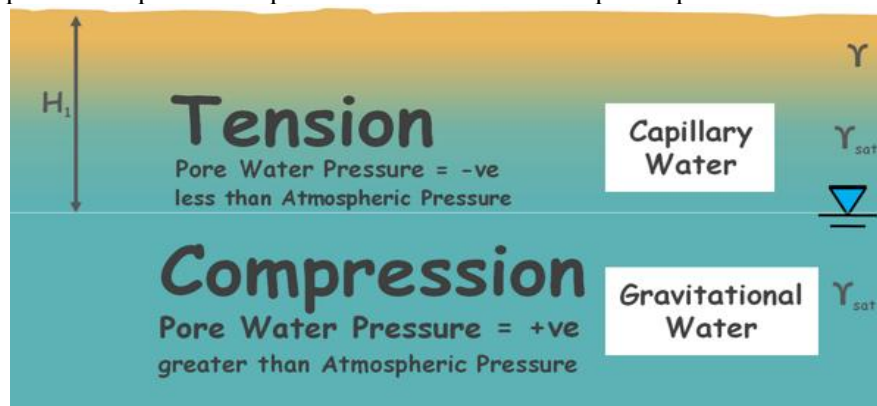
$$\sigma' = q + \gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{(sat)2} - \gamma_w) H_2''$$

we can write this quantity as submerged unit weight of soil

$$\sigma' = q + \gamma_1 H_1 + \gamma_2 H_2' + \gamma_2' H_2''$$

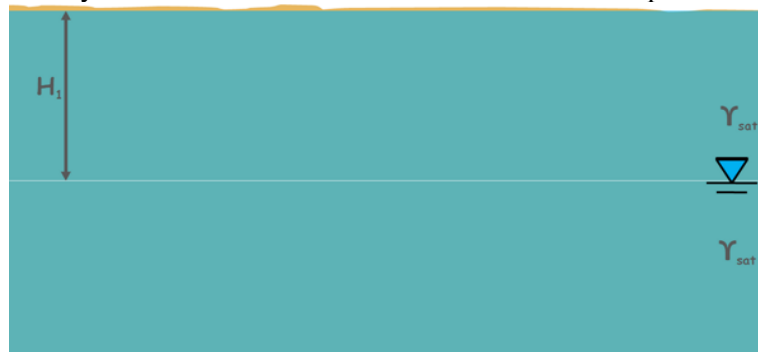
🌈 EFFECTIVE STRESS DUE TO CAPILLARY WATER EFFECT

That water molecules travel in upward direction in the soil even against the gravitational forces because of a phenomenon called the capillarity or capillary effect. Capillary effect is the ability of a liquid to flow in narrow spaces without the assistance of external forces. This risen water is called capillary water. to calculate the effective stress in soil at different depth when there is a capillary rise in the soil. So let us consider a soil mass of bulk unit weight γ and assume the water table lies at some depth H_1 below surface. Because of the water table soil becomes completely saturated and unit weight of soil becomes γ_{sat} . The water below water table is in compression because of the weight of water itself and the pore water pressure is positive that is above atmospheric pressure.



Soil above the water table becomes saturated because of capillary rise of water. So unit weight of this soil also becomes γ_{sat} . In case of capillary rise, water above the water table is in tension because it is pulled up by inter-molecular forces and the pore water pressure in this region is negative that is below atmospheric pressure.

Let us consider the capillary water has risen up to the surface and the soil becomes fully saturated up to the surface. We will try to determine the effective stress at different depths.



Let's begin with the surface. The total stress at the surface is zero because no load is present above this plane., $\sigma = 0$

No water is present above this plane, but capillary water is present here and it is in tension. So pore water pressure is not zero at the surface. It is unit weight of water γ_w multiplied by height up to which the capillary water has risen that is H_1 and it is negative as it is below atmospheric pressure.

$$u = -\gamma_w H_1$$

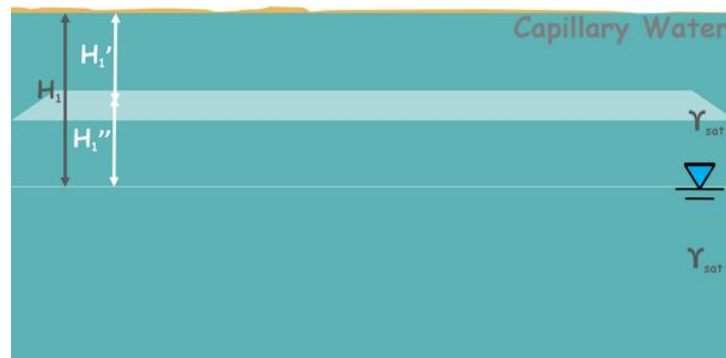
So effective stress at the surface of the soil is given as total stress minus pore water pressure.

$$\sigma' = \sigma - u$$

$$\sigma' = 0 - (-\gamma_w H_1)$$

And we can write it as this., $\sigma' = \gamma_w H_1$

If the capillary water is absent in the soil then at the surface pore water pressure would be zero and so would be effective stress. Now analyze the stresses below the ground surface at some plane in capillary saturation zone.



The total stress on this plane is because of this saturated soil above γ_{sat} multiplied by height of the soil above this plane say H_1' .

$$\sigma = \gamma_{sat} H_1'$$

Pore water pressure at this plane is negative because of capillary water present and is γ_w multiplied by the height of the capillary rise up to this plane say H_1'' .

$$u = -\gamma_w H_1''$$

We can also write this height as H_1 minus H_1'

$$H_1'' = H_1 - H_1'$$

So pore water pressure can be written as this:, $u = -\gamma_w (H_1 - H_1')$

We should note that water present above this plane does not cause to increase pore water pressure at this plane because this water is also capillary water and is in tension. It cannot compress the water below.

So effective stress at this surface can be written as this: $\sigma' = \sigma - u$

$$\sigma' = \gamma_{sat} H_1' - (-\gamma_w (H_1 - H_1'))$$

By solving we can write it is this.

$$\sigma' = \gamma_{sat} H_1' - \gamma_w H_1 + \gamma_w H_1'$$

$$\sigma' = (\gamma_{sat} - \gamma_w) H_1' + \gamma_w H_1$$

and this quantity can be written as submerged unit weight of soil. $\sigma' = \gamma' H_1' + \gamma_w H_1$

Now let's analyze the stresses at the plane present at water table.



The total stress on this plane because of the saturated soil above gamma sat H_1 .

$$\sigma = \gamma_{sat}H_1$$

Pore water pressure at this plane is zero because water on the water table remains at atmospheric pressure and water is neither in tension nor in compression., $u = 0$

So effective stress will be equal to total stress: $\sigma' = \sigma - u$

$$\sigma' = \gamma_{sat}H_1$$

Now move down the water table and analyze the stresses at a plane that is say at H_2 depth below water table.



The total stress on this plane is because of the saturated soil above the water table of height H_1 , gamma sat H_1 plus stress because of saturated soil below the water table of height H_2 , gamma sat H_2 .

$$\sigma = \gamma_{sat}H_1 + \gamma_{sat}H_2$$

Pore water pressure at this plane is compressive and is because of water below water table. Water above the water table has no effect under the water table.

So pore water pressure is positive gamma w H_2 .

$$u = \gamma_w H_2$$

Hence effective stress at this surface can be written as this:

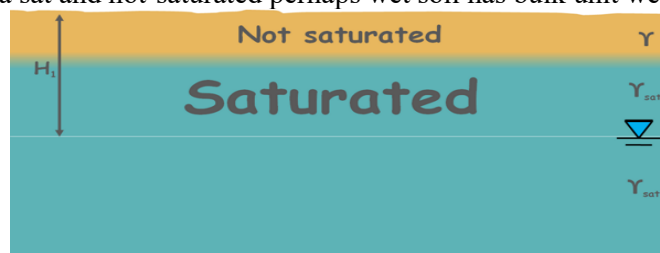
$$\sigma' = \sigma - u$$

$$\sigma' = \gamma_{sat}H_1 + \gamma_{sat}H_2 - \gamma_w H_2$$

And we can write this quantity as submerged unit weight of soil.

$$\sigma' = \gamma' H_1 + \gamma' H_2$$

Now let us take another case when capillary rise saturates the soil only up to some height above the water table and soil saturation does not reach up to the surface. Saturated part of the soil has unit weight gamma sat and not-saturated perhaps wet soil has bulk unit weight gamma.



Let's begin the analysis with the soil surface again.

The total stress at the surface is zero., $\sigma = 0$

pore water pressure is also zero as no water is present above the surface and no capillary water rose to the surface., $u = 0$

So effective stress at the surface is also zero.

$$\sigma' = \sigma - u$$

$$\sigma' = 0$$

Now let's analyze stresses at the plane, up to which the capillary water has saturated the soil.



The total stress on this plane is because of the soil above this plane, bulk unit weight γ and height of the soil above this plane say H_1' .

$$\sigma = \gamma H_1'$$

Pore water pressure at this plane is negative γ_w and height of this plane above water table say H_1'' .

$$u = -\gamma_w H_1''$$

So effective stress at this surface can be written as this: $\sigma' = \sigma - u$

$$\sigma' = \gamma H_1' - (-\gamma_w H_1'')$$

By solving we can write it is this., $\sigma' = \gamma H_1' + \gamma_w H_1''$

Now let's analyze the stresses on a plane at water table.



The total stress on this plane because of wet soil γ H_1' plus stress because of capillary saturated soil γ_{sat} H_1'' .

$$\sigma = \gamma H_1' + \gamma_{sat} H_1''$$

Pore water pressure at this plane is zero as water is at water table and at atmospheric pressure.

$$u = 0$$

So effective stress will be equal to total stress: $\sigma' = \sigma - u$

$$\sigma' = \gamma H_1' + \gamma_{sat} H_1''$$

Now move below the water table and analyze the stresses at this plane which say at H_2 depth.



The total stress on this plane is because of wet soil $\gamma H_1'$ plus because of capillary saturated soil above the water table $\gamma_{sat} H_1''$ plus because of saturated soil below the water table $\gamma_{sat} H_2$.

$$\sigma = \gamma H_1' + \gamma_{sat} H_1'' + \gamma_{sat} H_2$$

Pore water pressure at this plane is in compression and is because of water that is below water table $\gamma_w H_2$.

$$u = \gamma_w H_2$$

So effective stress at this surface can be written as this: $\sigma' = \sigma - u$

$$\sigma' = \gamma H_1' + \gamma_{sat} H_1'' + \gamma_{sat} H_2 - \gamma_w H_2$$

We can also write this as submerged unit weight of soil.

$$\sigma' = \gamma H_1' + \gamma_{sat} H_1'' + \gamma' H_2$$

SEEPAGE IN SOILS:

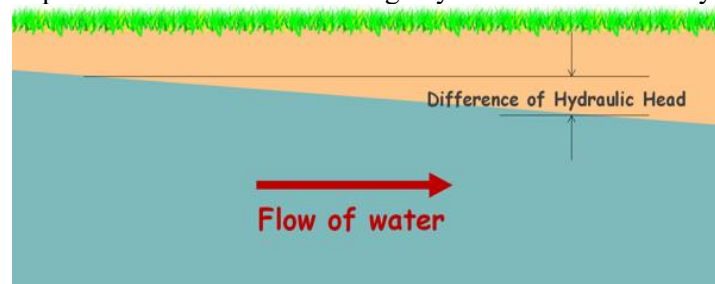
A Soil mass is the huge collection of small particles we call soil grains. These soil grains when depositing in a soil mass encloses empty space between them which we call voids.

The free water available under the ground moves inside the soil under the influence of gravity and it flows through these voids. This phenomenon of flow of water, movement of water through the soil is called seepage.

We know that water moves from high energy state to low energy state and in case of water these energy states are represented as hydraulic heads.

The hydraulic head is the amount of mechanical energy or liquid pressure available at any point in water above datum.

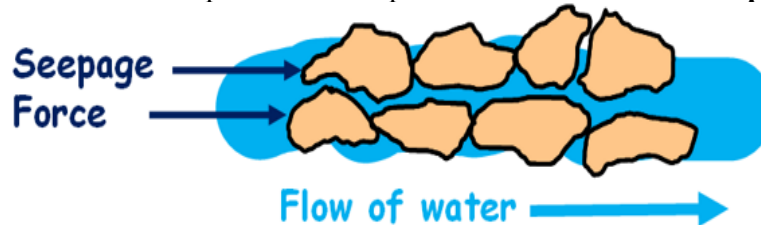
For the flow of water to take place in the soil also there should be a difference of hydraulic head between two points in space and water flows from high hydraulic head to low hydraulic head.



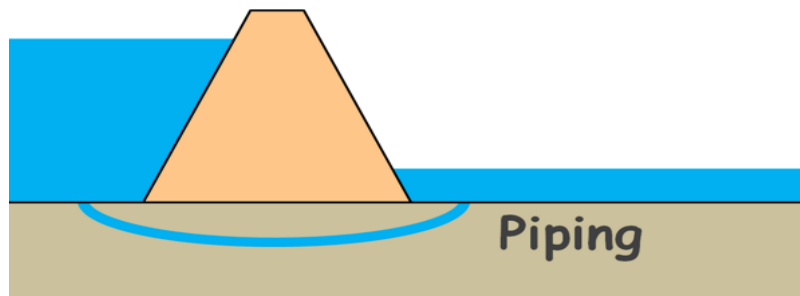
Hence we can say Seepage takes place when there is difference in hydraulic head.

Water does not flow if two points in the water have the same energy or have inadequate head difference. Water may flow even against the gravity if point A is at higher hydraulic head than that of point B.

As the water flows through the soil the surface of the soil particles offers a resistance to the flow, in turn water also pushes the soil particles and applies a drag on them in the direction of its motion. This drag increases the inter-particle forces. This force applied by the water on the soil particles is called the seepage force and because of it pressure developed in the soil is termed as **seepage pressure**.



When the water flows in the soil in horizontal direction, it exerts seepage force on the soil particles in the horizontal direction. If this exerted force is strong it has the capability to displace the soil particles in its way and particles may get carried away by the flowing water. This phenomenon leads to a very serious problem in the soil engineering, development of channels in soil and this phenomenon is called **piping**.



But here, we will discuss only the flow of water in the vertical direction. When water flows in vertical direction it has direct impact on effective stress acting on the soil particles.

Soil particles in a soil mass are burdened by the weight of everything above them which includes weight of all the soil particles above them, weight of water and weight of any structure if present above; and because of this weight these particles experience stress which we call total stress.

This total stress is shared by the two components in the soil mass. Part of the stress is shared by the water present in the voids of the soil structure and that is called pore water pressure.

And the other part of the stress is shared by soil grains by their grain to grain force transfer at their point of contact and that is called the effective stress. It is the part of the total stress that is acting on soil particles.

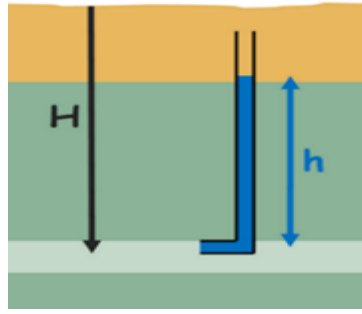
So total stress is equal to the sum of effective stress and pore water pressure.

$$\sigma = \sigma' + u$$

And the expression for effective stress is written as total stress minus pore water pressure.

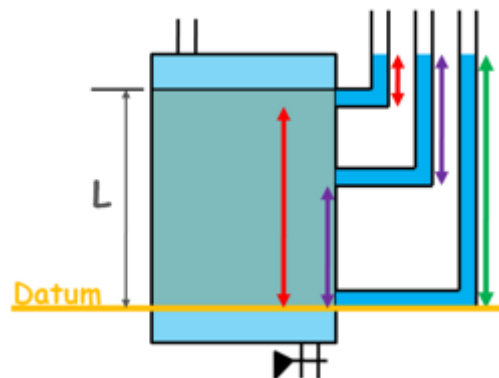
$$\sigma' = \sigma - u$$

This can be simplified to this: $\sigma = \gamma H$



To analyse the effect of seepage on effective stress, take a soil sample of say length L and cross-sectional area A . The sample is placed in a tank type container that has valves at its top and bottom. First keep the bottom valve closed and completely fill the container with water so as to submerge the soil. As the bottom valve is closed no water flows through the soil hence no seepage occurs. When water does not flow and remains stagnant in the soil the condition is known as hydrostatic condition.

Insert the standpipes at different depths in the soil and notice that water has risen in all the pipes up to the same elevation. But the height of rise is different for different pipes. The height of the water column represents the pore water pressure in the soil at that depth. So pore water pressure is different at different depths.



It is the total head, which is the sum of pressure head and elevation head, which is equal for all the depths in the soil in our hydrostatic condition, not the pore water pressure.

If we consider the bottom of the soil sample as our datum and measure elevations of the points of interest, we can see the sum of elevation head and pore water pressure is same at all depths that is total head is same; and when total head is same everywhere, water does not flow in any direction.

The total stress, pore water pressure and effective stress at different depths can be calculated for this no flow condition. Let's say height of the water above the soil sample is of height H_1 .

Total stress at the surface of the soil would be because of water above, $\sigma = \gamma_w H_1$

Pore water pressure will be γ_w and height of the water column in standpipe, H_1
 $u = \gamma_w H_1$

So, effective stress is zero., $\sigma' = \sigma - u$
 $\sigma' = 0$

Similarly, at the bottom of the soil sample total stress, because of water, $\gamma_w H_1$ and because of saturated soil, γ_{sat} multiplied by L , length of the soil sample.

$$\sigma = \gamma_w H_1 + \gamma_{sat} L$$

Pore water pressure because of the whole water γ_w multiplied by the height of the water column in the standpipe, which is length of the soil sample L plus height of the water above H_1 .

$$u = \gamma_w(L + H_1)$$

Hence effective stress is.

$$\sigma' = \sigma - u$$

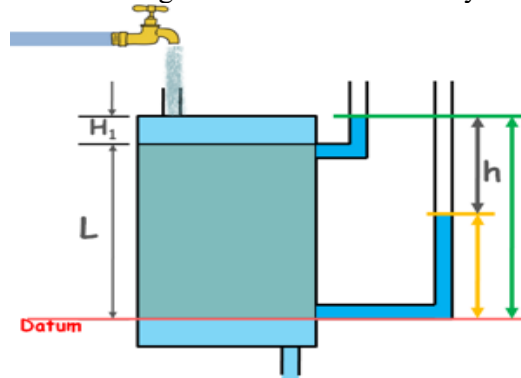
$$\sigma' = (\gamma_w H_1 + \gamma_{sat} L) - (\gamma_w (H_1 + L))$$

$$\sigma' = (\gamma_{sat} - \gamma_w) L$$

we can write it into submerged unit weight of soil, $\sigma' = \gamma' L$

These are the effective stresses at top and bottom of the soil when no seepage occurs in the soil.

Now open the valve at the bottom and let the water flow through the soil. The valve at the top is attached with a water source so as to maintain a constant level of water in the soil tank. This condition when water is not stagnant and it's flowing in the soil is known as hydro-dynamic condition.



the standpipes and notice that the water level in the tubes has been altered. It is not the same in all the tubes as it was earlier in case of hydrostatic condition. It is now different in different tubes.

The total head, that is pressure plus elevation head, at the top of the soil is higher and lower at the bottom of the soil. This difference of head at different depths in the soil clearly indicates that there is flow of water in the soil, as we know water flows from high head to low head.

Let's say the head difference between the entry head and exit head of water is small h .

Total stress distribution at different depths in the soil does not change by the flow of water as there is no change in the overburden pressure.

Total stress at the surface of the soil is stress because of water:

$$\sigma = \gamma_w H_1$$

Pore water pressure will be γ_w and height of the water column in standpipe H_1 .

$$u = \gamma_w H_1$$

So effective stress is zero. $\sigma' = \sigma - u$

$$\sigma' = 0$$

Total stress at the bottom of the soil is stress because of water $\gamma_w H_1$ plus stress because of saturated soil $\gamma_{sat} L$.

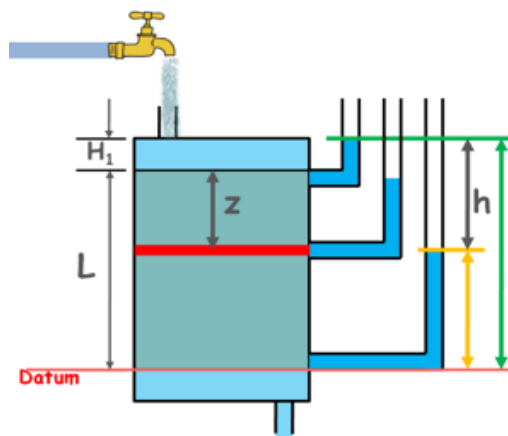
$$\sigma = \gamma_w H_1 + \gamma_{sat} L$$

Pore water pressure at the bottom plane is γ_w multiplied by the height of the water column in the standpipe, which is the total length of the soil and water ($H_1 + L$) minus head loss small h .

$$u = \gamma_w (H_1 + L - h)$$

$$\sigma' = (\gamma_{\text{sat}} - \gamma_w)L + \gamma_w h$$
$$\sigma' = \gamma' L + \gamma_w h$$

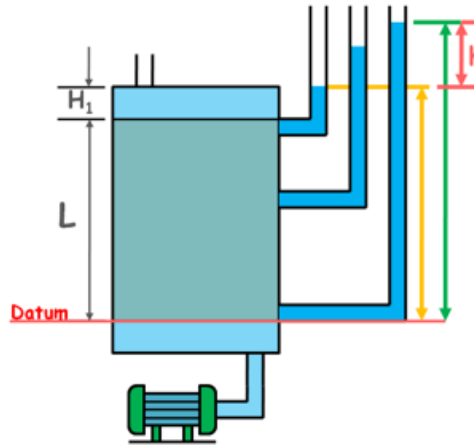
A diagram of a U-tube manometer. The left arm is filled with a blue liquid to a certain level. The right arm is partially filled with the same blue liquid, and the top part is empty. The height difference between the two liquid levels is labeled Δh . A horizontal double-headed arrow above the arms indicates the total length of the tube, labeled L . The background is divided into a green grassy top, an orange middle section, and a blue bottom section representing the liquid.

$$i = \frac{h}{L}$$
$$\sigma' = \gamma' L + iL\gamma_w$$

$$\sigma' = z\gamma' + \mathrm{i}z\gamma_{\mathrm{w}}$$

It clearly indicates that downward seepage in the soil increases the effective stress or inter-particle forces. This additional stress is due to seepage pressure which acts in the direction of flow of water. When water flows down the soil mass water tends to push soil particles downwards. Consequently soil particles push each other causing to increase the inter-particle forces and that is increase in effective stress.

So the increase in effective stress at any point below the surface at a depth z when water is flowing with a constant hydraulic gradient i can be written as $iz\gamma_w$.

Now let's reverse the situation and attach the water source to the bottom valve.



Now the flow of water in the sample is in upward direction, an upward seepage is taking place. Piezometric levels in the standpipes have changed and total head at the bottom of the soil has now become greater than the head at the top.

Let's say that the head difference between two ends of the soil are small h , so hydraulic gradient that is responsible for moving the water in the upward direction is

$$i = \frac{h}{L}$$

Let's analyse the stresses again. Effective stress at the top surface is again Zero. Now at the bottom surface total stress is because of this water is $\gamma_w H_1$ plus because of the saturated soil $\gamma_{sat} L$.

$$\sigma = \gamma_w H_1 + \gamma_{sat} L$$

Notice that total stress is unchanged.

Pore water pressure is γ_w multiplied by the height of the water column in the standpipe which is the total length of soil sample and water ($H_1 + L$) plus head difference small h .

$$u = \gamma_w (H_1 + L + h)$$

Hence effective stress can be written as this:

$$\sigma' = \sigma - u$$

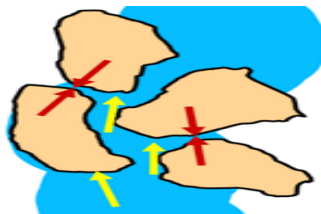
$$\sigma' = (\gamma_w H_1 + \gamma_{sat} L) - (\gamma_w (H_1 + L + h))$$

$$\sigma' = (\gamma_{sat} - \gamma_w) L - \gamma_w h$$

and it can be written in submerged unit weight of soil., $\sigma' = \gamma' L - \gamma_w h$

and head difference can be written as iL .

$$\sigma' = \gamma' L - iL\gamma_w$$



Now comparison with the previous conditions shows that the upward seepage in soil reduces the effective stress at the bottom of the soil by $iL\gamma_w$.

QUICK SAND CONDITION:

When water flows upwards in soil it tends to push the soil particles in upward direction. Consequently soil particles are lifted against gravity that causes to decrease the inter-particle forces and that we say decrease in effective stress.

Now if we notice that if head difference h keeps on increasing it may reach a condition when seepage pressure ($i\gamma_w L$) in the equation becomes equal to $\gamma' L$. At that point effective stress in the soil will become equal to zero.

$$\gamma' L - i\gamma_w L = 0$$

We can solve it for the hydraulic gradient at which effective stress in soil reaches to zero.

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

This hydraulic gradient is called **critical hydraulic gradient** and is written as i_{cr} .

When water flows in the soil in upward direction at the critical hydraulic gradient, the seepage force or drag exerted by water on soil particles, which pushes them up, completely balances the weight of these particles, and they seem to be suspended in water.

Soil in such state does not behave like a soil any more but behaves like a very viscous liquid. Soil loses all its shear strength and is no longer able to support any load.

This phenomenon is called **Quicksand**.

Note that Quicksand is not a type of sand, it is a hydraulic condition.

If we remember, we learned a relationship of soil parameters.

$$\gamma' = \frac{(G_s - 1)}{(1 + e)} \gamma_w$$

This relationship relates unit weight water with the properties of soil that are soil's unit weight, its void ratio and specific gravity of its solid particles. We can re-write it

$$\frac{\gamma'}{\gamma_w} = \frac{(G_s - 1)}{(1 + e)}$$

and substitute this as critical hydraulic gradient.

$$i_{cr} = \frac{(G_s - 1)}{(1 + e)}$$

It is an important relationship to be remembered. If we know specific gravity of soil particles in a soil mass and its void ratio we can calculate the critical hydraulic gradient for that soil, gradient at which the movement of water in upward direction in the soil is such that the soil becomes quick and loses all its shear strength and is not able to support any load.

Quicksand condition is also called boiling. This upward seepage force mainly affects fine sands and silts. It is less prominent in clay soils. Because First clays have poorly connected voids and flow of water is not as facilitative as in sand. And second clay particles possess cohesion which holds the grains together even at the critical hydraulic gradient.

Boiling does not occur in coarse sands and gravels either, because to lift gravel particles against gravity large water discharge is needed.

✚ Seepage Force

Seepage pressures can also be written as Seepage force that is the force applied by the flowing water to the soil structure. We know force is equal to pressure multiplied by area.

so seepage force is equal to seepage pressure multiplied by the area of the soil sample is capital A.

$$J = iL\gamma_w A$$

It is represented by capital letter J. But seepage force is usually expressed as force per unit volume.

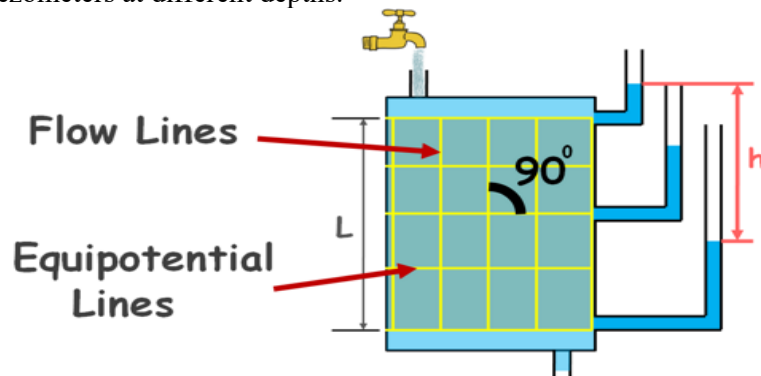
$$j = \frac{iL\gamma_w A}{AL} = i\gamma_w$$

✚ FLOW NET

As a soil engineer our aim is to analyse the flow of water inside the soil and try to obtain solutions to the engineering problems faced. One such problem is to estimate the quantity of water percolating via seepage through the soil under a dam. To solve such problems and to analyse multi-dimensional flow in soil, we make use of a concept called Flow net.

A flow net is a graphical representation of how the hydraulic energy is dissipated as water flows through a pervious medium.

Let us consider a soil sample of length L and put it into a glass cylinder. We attach the top of the cylinder with water source and let the water flow through the soil and let it exit from the bottom. We also attach few piezometers at different depths.



The flow of water occurs vertically downward under a head difference of say, h. Any water particle that enters into the soil at its top moves vertically downward and the path taken by this water particle can be represented by a line. That line is called flow line.

Similarly many particles will flow vertically downward and there will be infinite number of flow lines, but for our convenience we draw only few. These flow lines are also called Stream lines.

that at any level in the soil sample total head is same for all the points lying in that plane. So we can draw a line connecting all the points of equal head. Similarly at different levels such lines of equal heads can be drawn. These lines are known as equipotential lines. All the points on an equipotential line have the same energy.

Both of these lines, flow lines and equipotential lines, are crossing each other at right angles and we say they are orthogonal to each other. These lines form a kind of net and this net is called flow net. A flow net gives a pictorial representation of the path taken by water particles and the head variation along the path.

This is a very simple flow net and it is for a uni-directional flow in soil, but for multi-dimensional flow the flow net may be very complex. Generally the flow of water in soil is three dimensional and analysis of such flow is too complex and difficult. So we simplify the flow situations to two dimensional and analyse the flow.

Then how do we construct a flow net for a two dimensional flow? Well there are some methods for obtaining it.

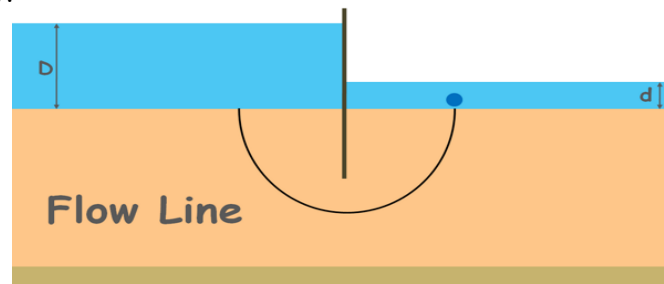
1. Analytical method
2. Graphical method
3. Electrical flow analogy

4. Capillary flow analogy
5. Sand model

GRAPHICAL METHOD

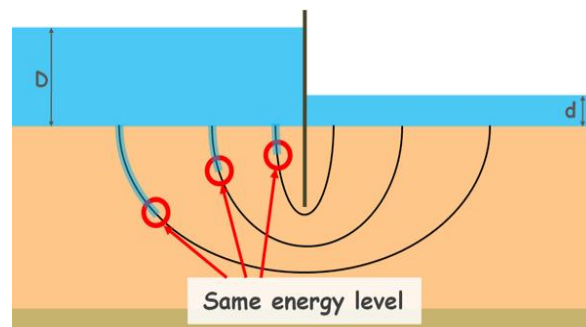
This is the most commonly used method of flow net construction because it is easy and it provides nearly accurate results. This method is one of the solutions to the Laplace equation.

Let us understand this method by drawing a flow net for a hydraulic condition. Let's take a soil mass of some thickness and it lies upon an impermeable strata. A sheet pile is driven into the soil up to some depth. The sheet has water on its one side of depth capital D and on another side small d. We can see there is imbalance of head on the different side of the sheet pile, so water will flow from high head to low head. But water cannot pass through this sheet pile so flow will take place through seepage via soil below.

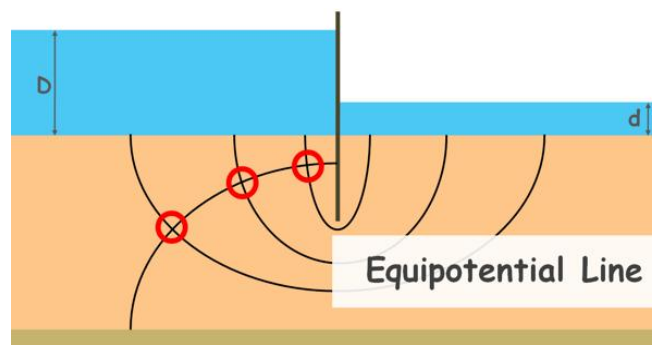


Let us consider a water molecule enters the soil at some point on the upstream surface, goes below the tip of the driven sheet and ends up at some point on the downstream surface. The flow path assumed by a water molecule is the flow line. Similarly many particles will leave the upstream and reach the downstream forming different flow lines.

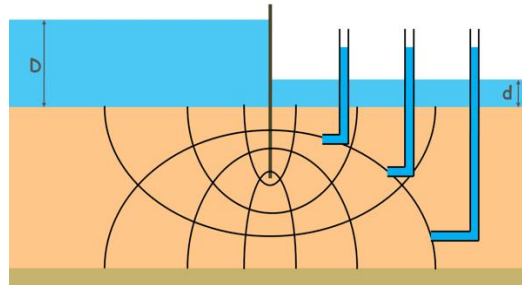
Note that each flow line begins from the upstream surface, which is at pressure $\gamma_w D$, and travels through the soil, constantly losing its energy and terminates at the downstream surface, which is at pressure $\gamma_w d$.



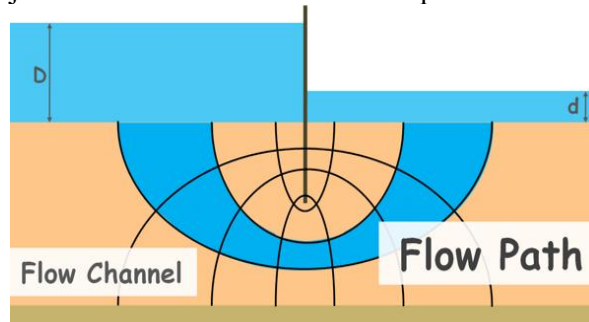
So, we can pick up certain points on the different flow lines where total energy lost is equal, or we can say points of same energy level. When we join such points together, the line so formed is an equipotential line.



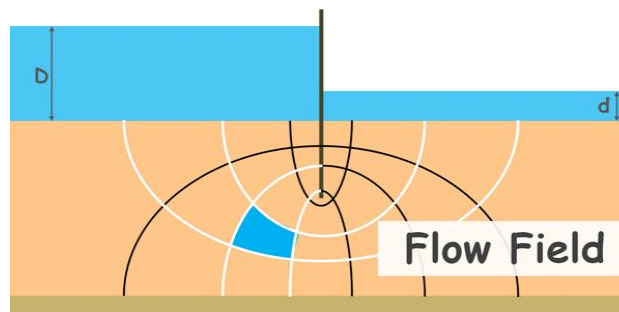
Similarly many different points of the same energy on different flow lines can be observed and many such equipotential lines can be drawn. If we insert piezometers into the soil at different points along an equipotential line we will notice water rises to the same elevation in all these piezometers.



The space between two adjacent flow lines is called the flow path or flow channel.

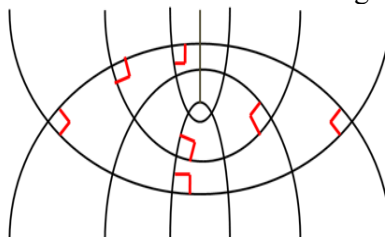


And the area enclosed between any two adjacent flow lines and adjacent equipotential lines is called Flow field.

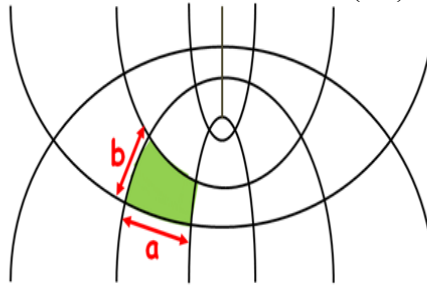


In a flow net we can draw many possible flow lines and equipotential lines, but first it is very inconvenient to draw too many of these lines and second drawing too many lines does not increase the accuracy of the calculations. So, only four or five flow channels are sufficient.

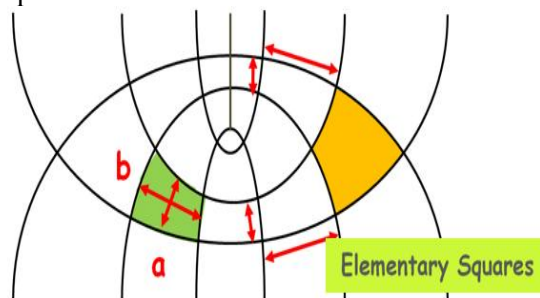
- To draw a meaningful graph we need to keep in mind certain characteristics of the flow net. First, two flow lines can never cross each other because flow in soil voids is considered laminar. Similarly no two equipotential lines can cross each other. Because if they cross at any point then the total head at that point will be two values, which is not possible.
- Second characteristic of the flow net is that flow lines and equipotential lines are orthogonal to each other. That means they should intersect each other at right angles.



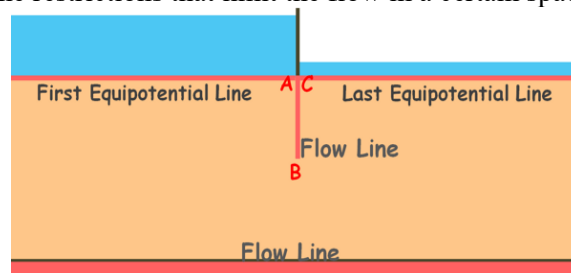
- Third, the ratio of the length and width of each flow field (a/b) should be constant.



Generally we take this ratio as unity for convenience. In other words, the flow net consists of approximate squares that are called elementary squares. In elementary squares the average distances between opposite sides are equal.



To construct a flow net we also need to identify the boundary conditions present for the flow. Boundary conditions are the restrictions that limit the flow in a certain space or area.

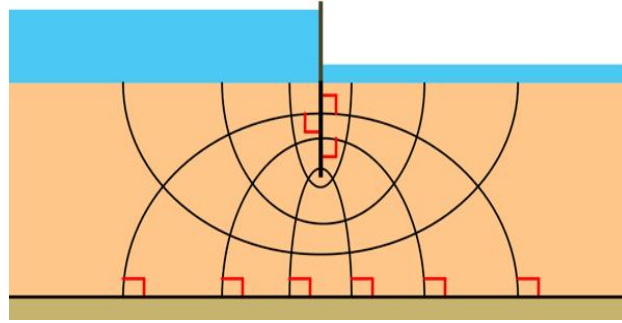


A flow net is unique for a given set of boundary conditions. If the geometry of the flow space changes, the boundary conditions will be changed and hence the flow net will be changed.

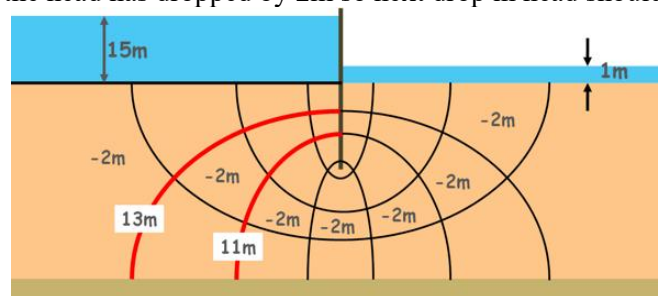
- Here, first boundary condition is the upstream surface, from where the flow starts and if we notice, it is the first equipotential line of our flow net as at every point on this line the total head is same.
- Second boundary condition is similar and that is the downstream surface, it is the last equipotential line of our flow net.
- The third boundary is the sheet pile. Water molecule cannot cross this sheet, it flows from a point which is near to the sheet on the upstream and moves vertically downward. and after crossing the sheet pile it vertically ascends. The sheet pile is also tracing the flow path of the molecule so this boundary, if we name it ABC, is a flow line.
- Fourth boundary is the bottom most impermeable surface. Water molecules cannot cross it. This line is also a flow line as the water will flow along this surface from one side to other.

While drawing a flow net we also need to keep in mind that no flow line can intersect the impermeable boundary as the impermeable boundary is itself a flow line. For the same reason all

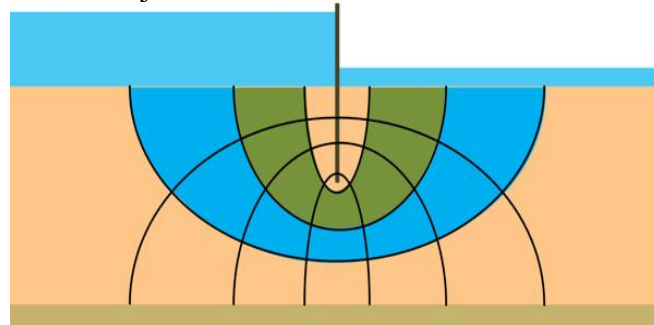
equipotential lines must meet the impermeable boundaries at right angles because impermeable boundaries are flow lines and flow lines meet equipotential lines at right angle.



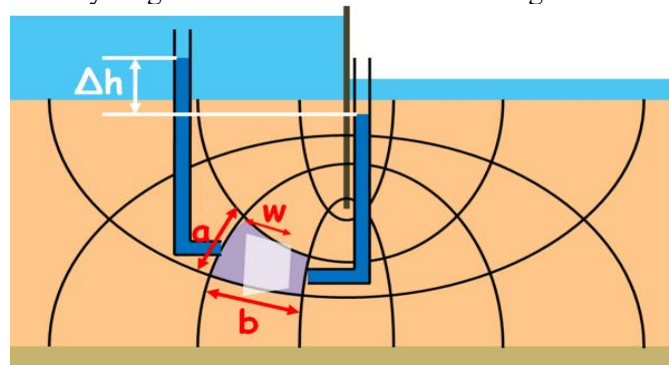
These equipotential lines are drawn in a way that, drop of head or loss of head between two adjacent equipotential lines is constant. Which means if head at the first equipotential line is 15m and after the first equipotential line the head has dropped by 2m so next drop in head should also be 2m and so on.



Also the discharge between two adjacent flow channels is constant.



Now that we have constructed the flow net its graphical properties can be used to calculate the seepage through soil. Let's say height of one flow field is a and length is b .



From Darcy's law the discharge through this flow field can be written as this, $q = kAi$
 k is the permeability of the soil. A is the area of the cross-section through which the water is flowing. Which is height of the flow field a and its width say w . But it is generally taken as 1 meter.

i is the hydraulic gradient which is head loss over the length of the flow field say Δh . So discharge through the flow field can be written as this.

$$\Delta q = k(a.1) \frac{\Delta h}{b}$$

Now Δh is the head loss through one flow field, or we can say drop of head or potential drop through one field is Δh , and let's say there are number of such drops, potential drops is N_d , then total potential drop or head loss through this whole flow channel is N_d times Δh .

$$h = N_d \Delta h$$

Let's substitute this in equation to find the discharge through one flow channel.

$$\Delta q = kh \frac{1}{N_d} \frac{a}{b}$$

Note that number of potential drops N_d is number of equipotential lines minus one.

$$N_d = \text{number of equipotential lines} - 1$$

Again this is the discharge through only one flow channel and if total number of flow channels in the flow net are say N_f then total discharge can be written as N_f times discharge through one flow field.

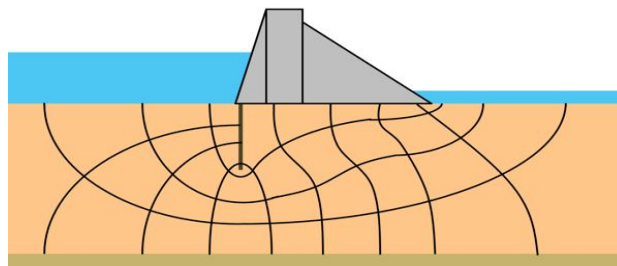
$$q = \Delta q = (kh \frac{1}{N_d} \frac{a}{b}) N_f$$

Remember that flow net is constructed by elementary squares. Hence $a/b = 1$.

So total discharge through soil can be written as this:

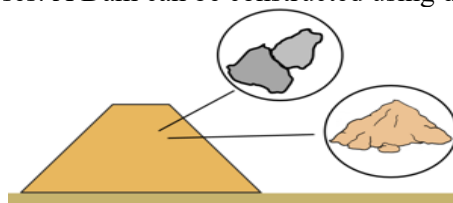
$$q = kh \frac{N_f}{N_d}$$

The ratio $[N_f/N_d]$ is called the shape factor. For a given set of boundary conditions the flow net is unique it is dependent on the boundary conditions. If boundary conditions changes, the flow net will also change.



But if we change the soil in which the water is flowing, the flow net will not change. Only the permeability of the soil k will change. Flow net will also remain unchanged even if the upstream and downstream water levels are reversed; only the direction of the flow will be reversed. Using the flow net to find solutions to our engineering problems such as the estimation of quantity of seepage losses from reservoirs, determination of seepage pressures, uplift pressures below dams, to check against the possibility of piping and many others.

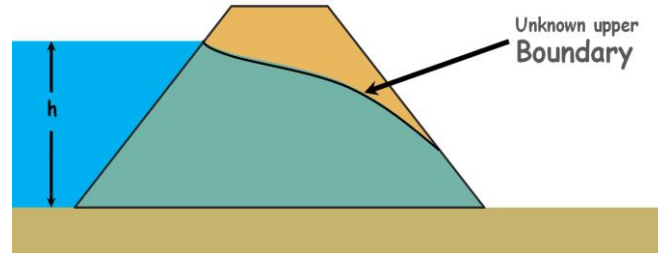
A dam is a barrier that restricts the flow of water and can be used to collect the water and this water may be utilized for many purposes. A Dam can be constructed using different kinds of materials.



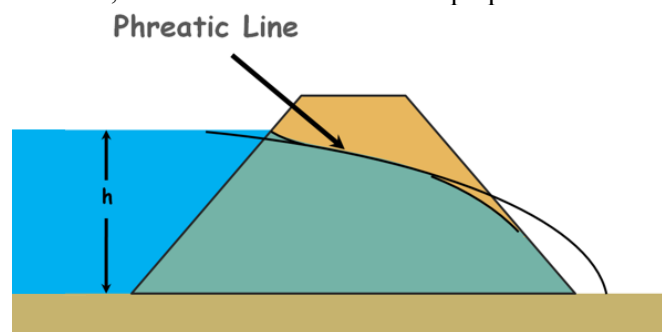
In earth dams the construction material is rocks and soil. The water that the dam holds flows from upstream through the voids present in the soil of the body of the dam to downstream by the action of hydraulic head difference. This phenomenon of movement of water through the soil is called seepage.

LOCATING THE PHREATIC LINE IN AN EARTHEN DAM

When there is no filter provided and seepage water exists from the downstream face of the dam. Let's assume an earth dam that retains the water up to the height h . Also assume that foundation of the dam is impermeable and water will only seep through the body of the dam. Seepage through an earth dam is the case of unconfined flow of which the upper boundary is not known, which is the top flow line.

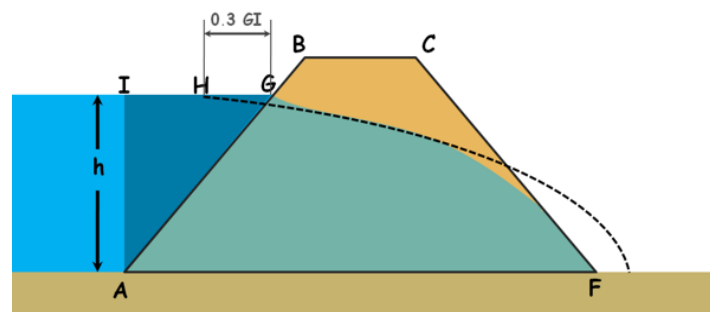


Casagrande observed that the shape of this top flow line, which is also called phreatic line, is close to the basic parabola for most of its length. So he recommended it can also be taken as basic parabola with necessary corrections at the upstream and downstream faces manually so that it conforms to basic conditions of the flow net, such as flow line should be perpendicular to the equipotential line.



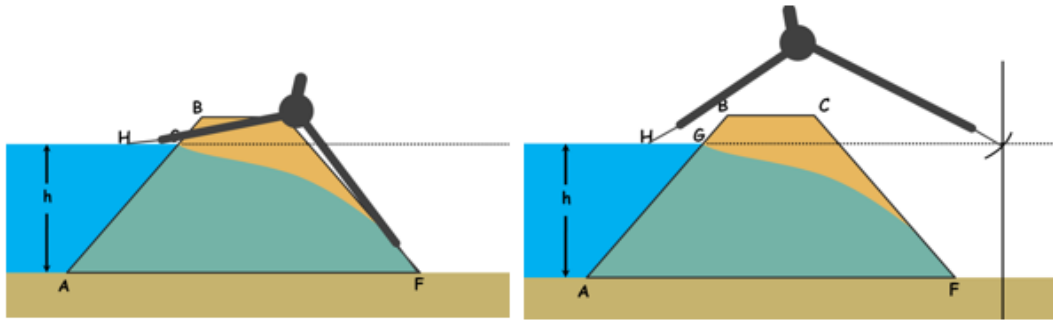
So to obtain the phreatic line first we need to draw the base parabola and for that we take the downstream point of the dam as the focus F of the parabola and also the origin. Also name few points in the diagram.

Casagrande also recommends that this seepage parabola curve starts from a point, say point H , on the upstream. If we take the horizontal projection of the wet upstream face of the dam on the water surface as GI , then the distance of this point H from the point G on the face of the dam, GH is given as 0.3 times GI .

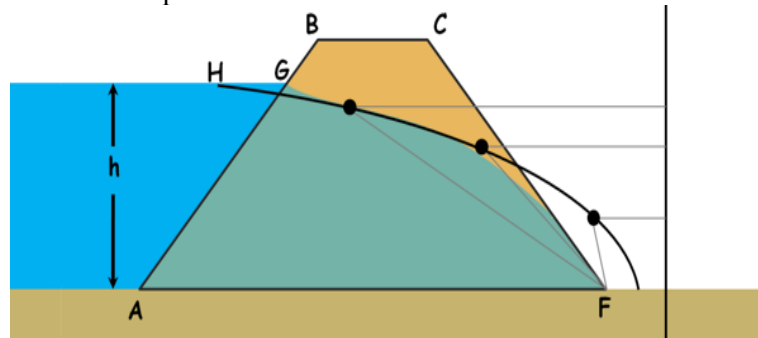


Now we can obtain the parabola's directrix using the property of the parabola that states, the distance of any point on the parabola from the directrix and its focus is equal. Point H is on the parabola and F is the focus of the parabola. Distance of this point H from the focus and parabola's directrix should be equal.

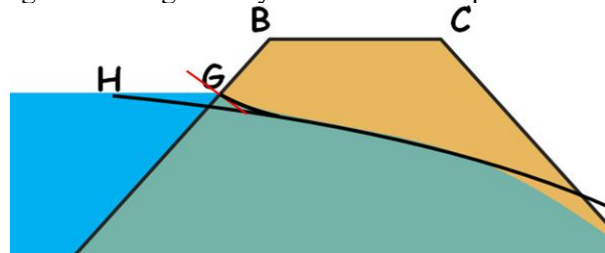
So to obtain the directrix we extend the upstream water level and using the compass of length HF make an arc that cuts this extension. From this point we draw a perpendicular to the base of the dam and we get our directrix.



Then we determine the intermediate points of the curve using the same parabola property. Then join all those points to find our base parabola.

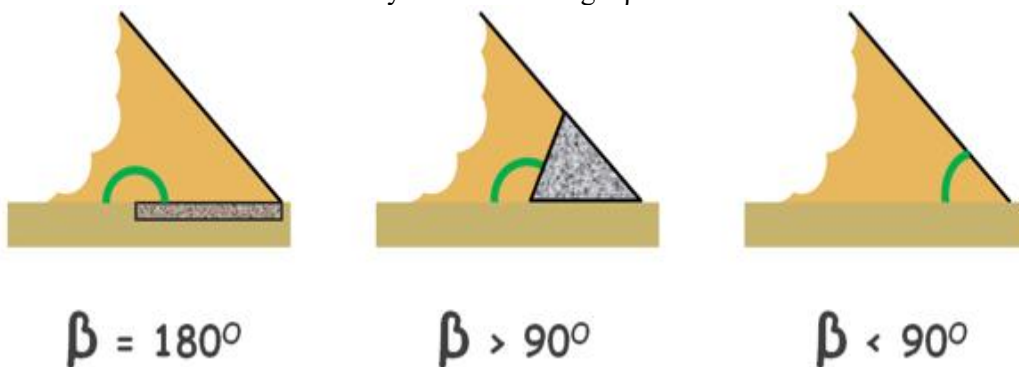


Now to draw the phreatic line from this parabola we will have to make some corrections. For the correction at the upstream we draw a perpendicular line to the upstream face AB at point G, from where the top flow line originates and gradually blend it into the parabola curve manually.



For the correction at the downstream we need to understand few things. When horizontal filter is not provided the seepage line cuts the downstream face of the dam and the downstream face acts as the discharge face. When the seepage water exits from the downstream face of the dam it may carry away the dam material with it, which may cause to create pipe like openings. This phenomenon, called piping, may cause the failure of the downstream slope or the whole dam.

The downstream face of the dam is inclined with horizontal. This inclination of the discharge face is described relative to the bottom boundary flow line as angle β and is measured clockwise.



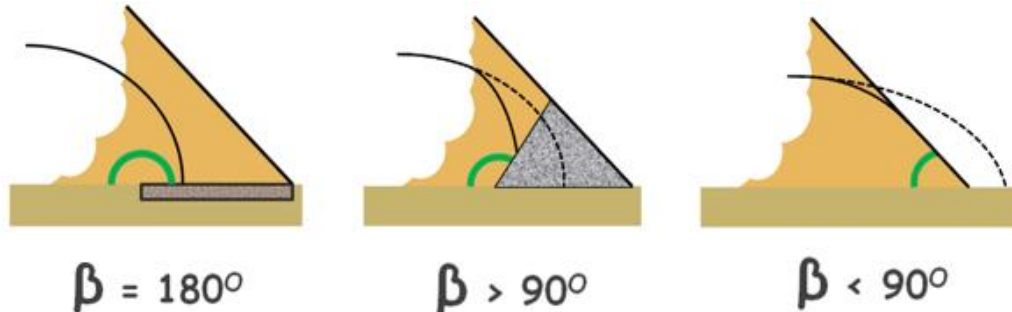
In case of the horizontal filter, the angle between bottom flow line and discharge face is this and β is 180° . In case of discharge face being a rock toe this angle β is greater than 90° . In case of discharge face being the downstream face the angle β will be less than 90° .

Phreatic line meets differently to the different types of discharge faces.

When β is 180° phreatic line meets the discharge face exactly as the base parabola.

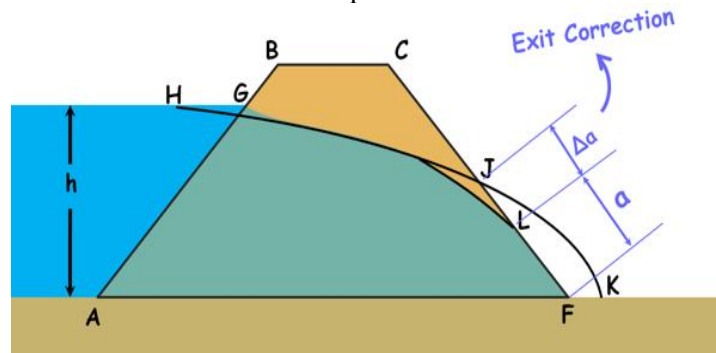
When β is greater than 90° phreatic drops vertically on the discharge face.

When β is less than 90° the phreatic line meets the discharge face tangentially and extends towards the toe of the dam.



So in both of the last two cases an additional correction at exit is required.

When β is less than 90° the basic parabola goes outside the downstream face. Let's say the base parabola cuts the dam at the downstream at some point J and meet the horizontal at point K.



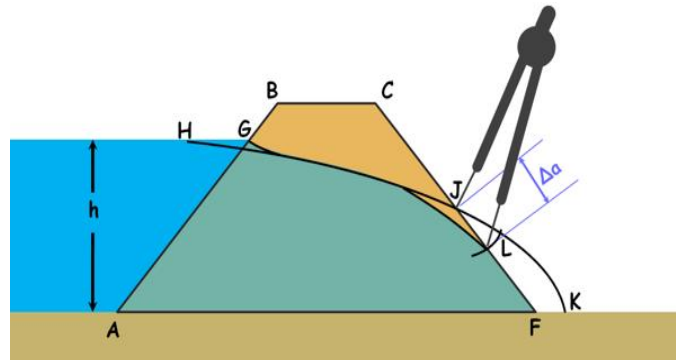
The actual phreatic line in this case curves to meet the downstream discharge face tangentially at some point say L below the point J.

The deviation JL is the distance by which the basic parabola has to be shifted to obtain the actual phreatic line. It is called the exit correction and it is denoted as Δa . And the distance JF is written as $a + \Delta a$.

β	$\frac{\Delta a}{a + \Delta a}$
30°	0.36
60°	0.32
90°	0.26
120°	0.18
150°	0.10
180°	0.0

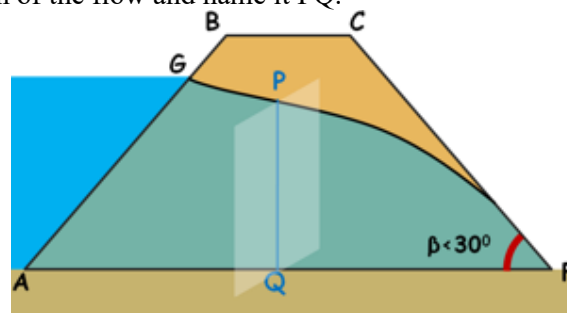
What is the value of Δa . Casagrande observed this Δa varies with the downstream slope angle β . So he gave a chart for the exit correction Δa the distance by which the basic parabola is shifted to locate the point where the actual seepage line cuts the discharge face.

In the chart we can notice that the exit correction Δa is zero when the angle β is 180° . If we remember we also did not have to apply the exit correction in the case of horizontal filter. Once we obtain Δa an arc of Δa length is cut on the downstream face and the base parabola curve is joined at this point to obtain the actual phreatic line.



So this is how we create the phreatic line for an earth dam when seepage water exits from the downstream face of the dam. this chart is applicable only when the downstream slope angle β is greater than or equal to 30° ($\beta \geq 30^\circ$).

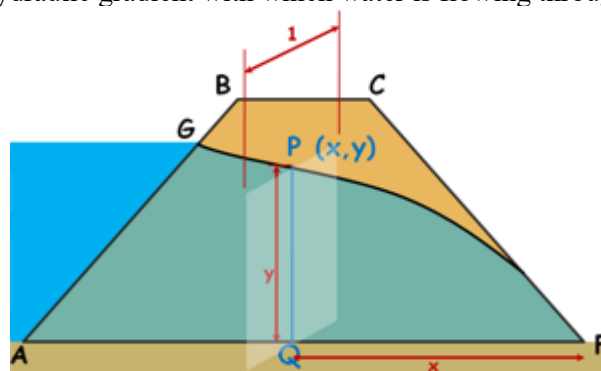
Now Let's try to estimate the seepage through an earth dam with discharge angle less than 30°
Let us consider any section of the flow and name it PQ.



As we know the flow of free water through the soil is governed by the Darcy's law, we can write discharge through this section as

$$q = kAi$$

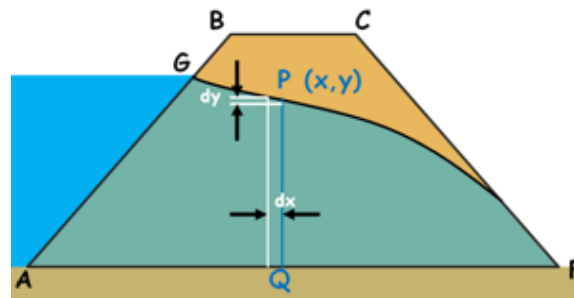
k is the permeability of the dam material., A is area of the cross-section through which seepage water is passing. And i is the hydraulic gradient with which water is flowing through this section.



Say co-ordinates of point P is x,y then height of this section which is at distance x from origin is y. lets calculate the discharge for unit width of the dam. So area of this section can be written as this

$$q = kAi$$

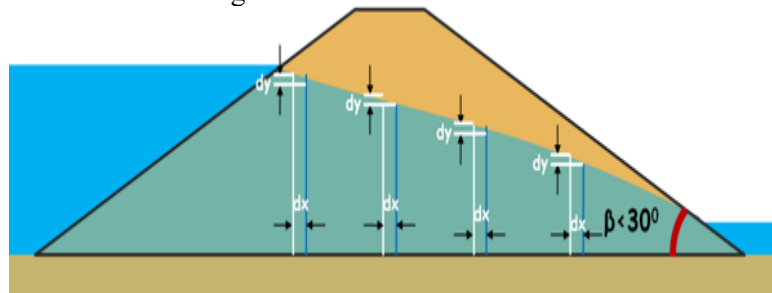
$$= \gamma(1)$$



Let's say a slight distance dx away from this section we observe a head difference of dy . So hydraulic gradient causing the flow at this section can be written as change in head over the distance over which the change occurred.

$$q = kAi = \frac{dy}{dx}$$

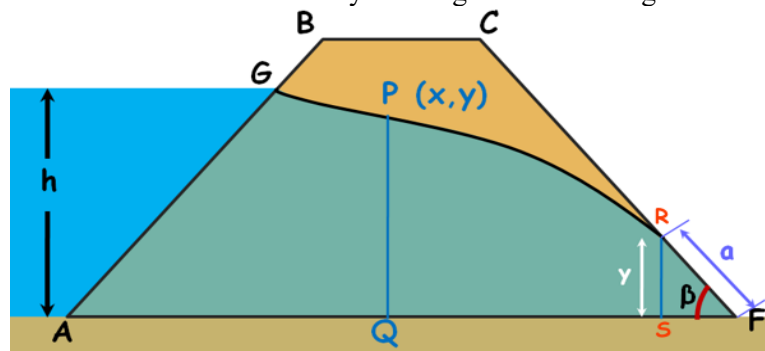
According to Dupuit's assumption, hydraulic gradient for a dam having discharge face angle less than 30° is equal to the dy/dx . So we assume that the hydraulic gradient is same at all the locations throughout the dam. Which means for $\beta < 30^\circ$ the length of the flow becomes so long that it roughly becomes a straight line with uniform gradient.



So we can write discharge through this section as this

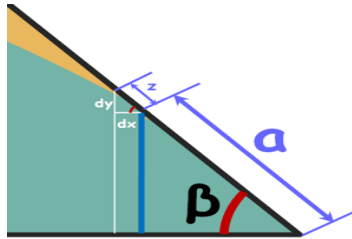
$$q = ky \frac{dy}{dx}$$

Discharge through any section of the dam is equal to the discharge through the section where the phreatic line touches the downstream face because whatever amount of water coming from any section is going through this section as foundation is impermeable. At this section because of the geometry it is easier to determine the value of hydraulic gradient and height of the cross-section.



now from the triangle RSF height of section RS, that is y , can be written as $a \sin \beta$

$$q = ky \frac{dy}{dx} = a \sin \beta$$



Now at dx distance away from this section change in head is dy . So by this triangle we can determine the value of dy and dx as $z \sin \beta$ and $z \cos \beta$

$$dy = z \sin \beta$$

$$dx = z \cos \beta$$

So Hydraulic gradient can be written as, $i = \sin \beta / \cos \beta = \tan \beta$

So discharge through this section can be written as

$$q = ky \frac{dy}{dx} = \frac{\sin \beta}{\cos \beta} \\ = a \sin \beta$$

$$q = k(a \sin \beta) \tan \beta$$

Now in this equation only unknown is a and once we know the a we can calculate the discharge through the dam. a can be found out by equating the discharge through any section of the dam and discharge through the section RS of the dam.

$$q = ky \frac{dy}{dx} = q = k(a \sin \beta) \tan \beta$$

By integrating and applying simple mathematics we arrive at a very simple equation which gives the value of a .

$$a = \frac{d}{\cos \beta} - \sqrt{\frac{d^2}{\cos^2 \beta} - \frac{h^2}{\sin^2 \beta}}$$

Now let's try to estimate the seepage through an earth dam with discharge angle greater than 30° but less than 60° . When β was less than 30° we calculated discharge through the dam using Dupuit's assumption that hydraulic gradient will be dy/dx .

But when β is greater than 30° and less than 60° Casagrande suggested that hydraulic gradient is given by

$$i = \frac{dy}{ds}$$

Where distance s is measured along the curve. Again After some easy calculations we find the discharge through the dam as this

$$q = k a \sin^2 \beta$$

And the value of a in this case as this

$$a = \sqrt{d^2 + h^2} - \sqrt{d^2 - h^2 \cot^2 \beta}$$

To arrive at these equations we use some approximations and for discharge angles greater than 60° the error because of those approximations becomes large. So we do not use this method for β greater than 60° .