: 2. Magnetostatics :

Topics: Biot-Savart's law, Ampere's Circuital law and Applications, magnetic flux density, Maxwell's Two Equations for magnetostatic Fields, Magnetic scalar and Vector potentials, Forces due to magnetic fields, Ampere's Force law, Illustrative problems. Maxwell's Equations (Time varying Fields): Faraday's law and Transformer EMF, inconsistency of Ampere's law and Displacement current density, Maxwell's Equations in Different final Forms and word statements, Conditions at a boundary Surface: Dielectric-Dielectric and Dielectric-Conductor Interfaces, Illustrative problems.

A definite link between electric and magnetic fields was established by Oersted in 1820, A Danish professor of physics, after 13 years of frustrating efforts discovered that electricity could produce magnetism. As we have noticed, an electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced. A magnetostatic field is produced by a constant current flow (or direct current). This current flow may be due to magnetization currents as in permanent magnets, electron-beam currents as in Vacuum tubes, or Conduction currents as in current-carrying wires.

Some birds, bees and certain animals are blessed with magnetic sense but man can sense magnetic fields only with a Compass.

Magnetoreception is a sense which ~~that~~ allows an organism to detect a magnetic field to perceive direction, altitude or location. This sensory modality is used by a range of animals for orientation and navigation, and as a method for animals to develop regional maps.

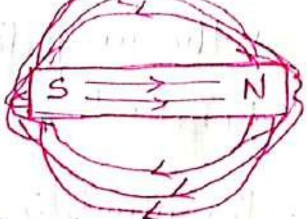
Ex: Bacteria, snails, frogs, lobsters.

Applications of magnetostatic fields:

1. Motors
2. Transformers
3. Microphones
4. Compasses
5. Telephone bell ringers
6. Television focusing controls
7. Advertising displays.
8. Magnetically levitated high speed vehicles
9. Memory stores
10. Magnetic separators.

Fundamentals of steady magnetic fields:

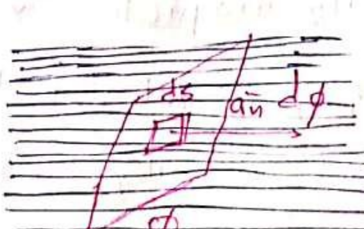
Steady magnetic fields are also called static magnetic fields or magnetostatic fields. These are produced by a magnet or by a current element. It is well known that loadstone is a natural magnet. But it is a fairly weak magnet. Strong magnets are made of iron, nickel, cobalt or steel. The two opposite ends of a magnet are called its poles.



If a magnet is floated freely, one pole will point towards the north pole and is called the north pole of the magnet. The other pole is the south pole.

Magnetic flux and Flux Density:

Magnetic flux gives the distribution of magnetic field passing through a given surface. In other words



the total number of magnetic lines of force passing through a given surface is called magnetic flux. It is denoted by the symbol ϕ and unit is weber.

Magnetic flux density is defined as the magnetic flux crossing a unit area normal to the direction of the magnetic flux

$$\bar{B} = \frac{d\phi}{ds}\,\bar{a_n} \quad wb/m^2 \text{ or Tesla}$$

$$\phi = \int_s \bar{B}\cdot d\bar{s}$$

B is also defined as $\bar{B} = \mu \bar{H}$

$\mu_r = 0.99999$ for silver, lead, copper, water (Diamagnetic)

$\mu_r = 1$ for Air, aluminium, tungsten, platinum (paramagnetic)

$\mu_r = 250$ cobalt, $= 600$ Nickel — Ferromagnetic

$\bar{H}$ - Magnetic field (A/m)

μ - permeability of the medium ($\mu_0 \mu_r$) (H/m)

μ_0 - permeability of free space $4\pi \times 10^{-7}$ H/m

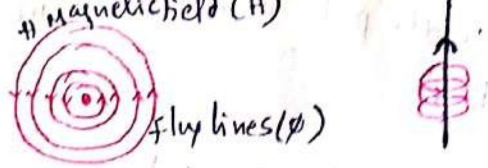
μ_r - Relative permeability of the medium.

Magnetic field Intensity or Magnetic field strength:

The magnetic field intensity at any point in a magnetic field is defined as the magnetic force experienced by a unit pole at that point. H is also called magnetic field strength

H is a vector quantity denoted by H. Units are N/wb or A/m

$\bar{H}$ is independent of the medium's permeability



Problem: If the magnetic flux density in a medium is given by $B = \frac{1}{\rho} \cos\phi \, \bar{a}_\rho$. What is the flux crossing the surface defined by $-\pi/4 < \phi \le \pi/4$, $0 \le z \le 2$ m

Solution:

$$B = \frac{1}{\rho} \cos\phi \, \bar{a}_\rho \qquad ds = \rho \, d\phi \, dz \, \bar{a}_\rho$$

$$\phi = \oint_c \bar{B} \cdot d\bar{s}$$

$$= \int_{z=0}^{2} \int_{\phi=-\pi/4}^{\pi/4} \frac{1}{\rho} \cos\phi \, \bar{a}_\rho \cdot \rho \, d\phi \, dz \, \bar{a}_\rho = \int_0^2 \int_{\phi=-\pi/4}^{\pi/4} \cos\phi \cdot d\phi \, dz$$

$$= [+\sin\phi]_{-\pi/4}^{\pi/4} [z]_0^2 = \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right](2) = 2\sqrt{2}$$

$$= 2 \times 1.414 = 2.828 \text{ weber}$$

Scanned by CamScanner

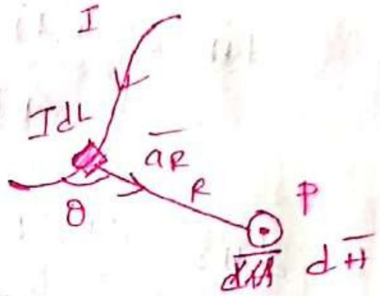
Problem: Given $\bar{H} = 4\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z$ A/m at point in free space. what is the flux density.

solution: $\bar{H} = 4\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z$ A/m

$$\bar{B} = \mu\bar{H} = \mu_0\bar{H} = 4\pi\times10^{-7}\left[4\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z\right]$$

$$= 5.026\bar{a}_x + 2.513\bar{a}_y - 3.767\bar{a}_z \ \mu Wb/m^2$$

Biot-Savart's law:



Consider a current-carrying Conductor producing a steady magnetic field around it. Let dl be a differential length on the conductor with current element IdL. let us consider a point p at a distance R from the current element. as shown in figure

Biot-savart's law states that the magnetic field intensity dH produced at a point p due to the differential-current element flowing through a conductor is proportional to the product of the current I and the differential conductor length dl, called current element (IdL). and the sine angle between the direction of the element and line joining point p to the current element. It is also inversely proportional to the square of the distance R between the current element and the point 'P'.

$$dH \propto IdL$$

$$dH \propto \sin\theta$$

$$dH \propto 1/R^2$$

$$dH \propto \frac{IdL\sin\theta}{R^2}$$

$$d\bar{H} = \frac{I d\vec{L} \sin\theta}{4\pi R^2}$$

from the definition of the cross product,
this can be written as
$$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{\bar{R}}{R}$$

$$d\bar{H} = \frac{I d\bar{L} \times \bar{a}_R}{4\pi R^2} \quad A/m$$

$$\cancel{\bar{a}_R} = \frac{\cancel{|\bar{R}|}}{\cancel{|\bar{R}|}} = \cancel{|\bar{R}|}$$

$$d\bar{H} = \frac{I d\bar{L} \times \bar{a}_R}{4\pi R^3}$$

$$\bar{H} = \oint d\bar{H} = \oint \frac{I d\bar{L} \times \bar{a}_R}{4\pi R^2} = \oint \frac{I d\bar{L} \times \bar{R}}{4\pi R^3} \quad A/m \text{ or } N/weber$$

$$\bar{B} = \mu \bar{H} = \frac{\mu}{4\pi} \oint \frac{I d\bar{L} \times \bar{a}_R}{R^2} = \frac{\mu}{4\pi} \oint \frac{I d\bar{L} \times \bar{R}}{R^3} \quad wb/m^2 \text{ or Tesla}$$

Biot-savart's law is also called Ampere's law for a current element.

Problem: A steady differential current element $10^{-3}\bar{a}_z$ A-m is located at the origin in free space. (a) Find the magnetic field flux density $\bar{B}$ at i) $(1,0,0)$ ii) $(0,1,0)$ iii) $(0,0,1)$

Solution: Given current element $I d\bar{L} = 10^{-3} \bar{a}_x$

i) The differential magnetic field density $\bar{B}$ due to $I d\bar{L}$ located at 'origin' $(0,0,0)$ is

$$d\bar{B} = \mu_0 \frac{I d\bar{L} \times \bar{R}}{4\pi R^3} \qquad \bar{R} = (1,0,0) - (0,0,0) = \bar{a}_x$$

$$= \frac{\mu_0}{4\pi} \frac{10^{-3}\, \bar{a_z} \times \bar{a_x}}{(1)^3} = 10^{-10} \bar{a_y} \text{ wb/m}^2 \text{ Tesla}$$

ii) The magnetic flux density $d\bar{B}$ due at point (0,1,0) due to current element IdL at (0,0,0) is

$$d\bar{B} = \frac{\mu_0\, I d\bar{L} \times \bar{R}}{4\pi R^3}$$

$$\bar{R} = (0,1,0) - (0,0,0) = \bar{a_y} = (0,1,0)$$

$$= \frac{\mu_0}{4\pi} \frac{10^{-3}\, \bar{a_z} \times \bar{a_y}}{(1)^3} = -10^{-10} \bar{a_x} \text{ wb/m}^2$$

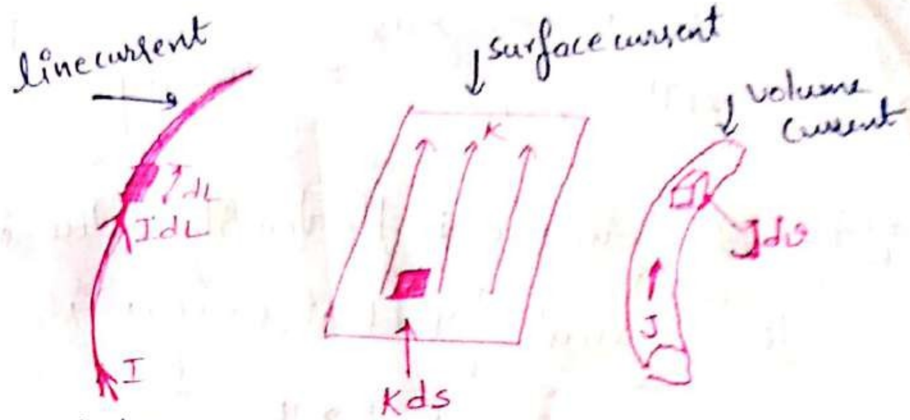
iii) The magnetic flux density $d\bar{B}$ at point (0,0,1) due to current element IdL at (0,0,0) is

$$d\bar{B} = \frac{\mu_0\, I d\bar{L} \times \bar{R}}{4\pi R^3}$$

$$\bar{R} = (0,0,1) - (0,0,0) = \bar{a_z} = (0,0,1)$$

$$= \frac{\mu_0}{4\pi} \frac{10^{-3}\, \bar{a_z} \times \bar{a_z}}{(1)^3} = 0 \text{ wb/m}^2$$

Note: It is observed that the magnetic field is always perpendicular to the current-carrying element.



Current distributions

K – is the surface current density A/m

J – is the volume current density A/m^2

$$I dL = K ds = J dv$$

$$\bar{H} = \int_L \frac{I dL \times \bar{a}_R}{4\pi R^2} = \int_L \frac{I d\bar{L} \times \bar{R}}{4\pi R^3} \text{ (line current)}$$

$$\bar{H} = \int_S \frac{K d\bar{s} \times \bar{a}_R}{4\pi R^2} = \int \frac{K d\bar{s} \times \bar{R}}{4\pi R^3} \text{ (surface current)}$$

$$\bar{H} = \int_V \frac{J d\bar{v} \times \bar{a}_R}{4\pi R^2} = \int \frac{J d\bar{v} \times \bar{R}}{4\pi R^3} \text{ (volume current)}$$

problem: Given magnetic flux density $B = \rho a_\phi$, find the total flux crossing the surface $\phi = \pi/2$, $1 \le \rho \le 2$, $0 \le z \le 5m$

solution:

$$\phi = \oint_S \bar{B} \cdot d\bar{s} \qquad d\bar{s} = d\rho\, dz\, \bar{a}_\phi$$

$$= \int_{z=0}^{5} \int_{\rho=1}^{2} \rho\, d\rho\, dz = \left[\frac{\rho^2}{2}\right]_1^2 [z]_0^5$$

$$= \cancel{1.5} \left[\frac{4}{2} - \frac{1}{2}\right][5-0] = 7.5 \text{ weber}$$

problem: A circular coil of radius 2.0 cm is in a magnetic flux density of $10\, wb/m^2$. If the plane of the coil is perpendicular to the field, determine the total flux around the coil.

solution

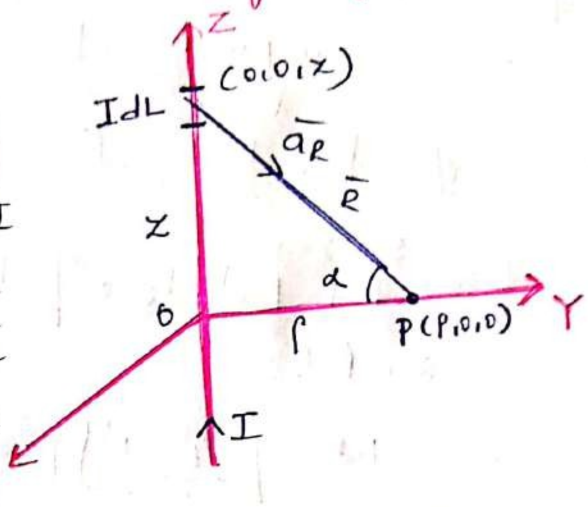
$B = 10\, wb/m^2$

$S = \pi r^2 = \pi (2 \times 10^{-2})^2 = 4\pi \times 10^{-4}\, m^2$

$\phi = B\, S = 10 \times 4\pi \times 10^{-4} = 12.56 \times 10^{-3} = 12.56\, m$ weber

Magnetic field Intensity due to an Infinitely long Conductor

Consider an infinitely long Conductor carrying current I along the z-axis. Let there be a current element $I\bar{dL}$ on the Conductor at distance 'z' from the origin.



Also let us consider a point P on the y-axis at a distance ρ from the origin as shown in figure.

In cylindrical coordinates, the position of the current element is $z\bar{a_z}$ and the position vector of the point P is $(\rho,0,0)$

$$\bar{R} = (\rho,0,0) - (0,0,z) = \rho\bar{a_\rho} - z\bar{a_z}$$

$$I\bar{dL} = I dz\, \bar{a_z} \qquad R = |\bar{R}| = \sqrt{\rho^2+z^2}$$

$$\bar{a_R} = \frac{\rho\bar{a_\rho} - z\bar{a_z}}{\sqrt{\rho^2+z^2}}$$

The magnetic field Intensity at point P is

$$\bar{H} = \int \frac{I\bar{dL}\times\bar{a_R}}{4\pi R^2} = \int \frac{I dL\times \bar{R}}{4\pi R^3}$$

$$I\bar{dL}\times\bar{R} = I dz\, \bar{a_z} \times \frac{(\rho\bar{a_\rho} - z\bar{a_z})}{\sqrt{\rho^2+z^2}} = \qquad \begin{aligned}\bar{a_z}\times\bar{a_\rho} &= \bar{a_\phi} \\ \bar{a_z}\times\bar{a_z} &= 0\end{aligned}$$

$$\bar{H} = \int_{-\infty}^{\infty} \frac{I\rho\, dz\, \bar{a_\phi}}{4\pi(\rho^2+z^2)^{3/2}}$$

To evaluate the integration

let $z = \rho \tan\alpha$ $\quad z=-\infty \quad \alpha = -\pi/2$

$dz = \rho \sec^2\alpha \, d\alpha \quad z = \infty \quad \alpha = \pi/2$

$$= \int_{-\pi/2}^{\pi/2} \frac{I\rho \cdot \rho \sec^2\alpha \, d\alpha \, \bar{a}_\phi}{4\pi(\rho^2+\rho^2\tan^2\alpha)^{3/2}} = \int_{-\pi/2}^{\pi/2} \frac{I}{4\pi\rho} \bar{a}_\phi \cos\alpha \, d\alpha$$

$$= \frac{I}{4\pi\rho} \bar{a}_\phi \left[\sin\alpha\right]_{-\pi/2}^{\pi/2} = \frac{I}{4\pi\rho} \bar{a}_\phi [1+1] = \frac{I\bar{a}_\phi}{2\pi\rho} \text{ A/m}$$

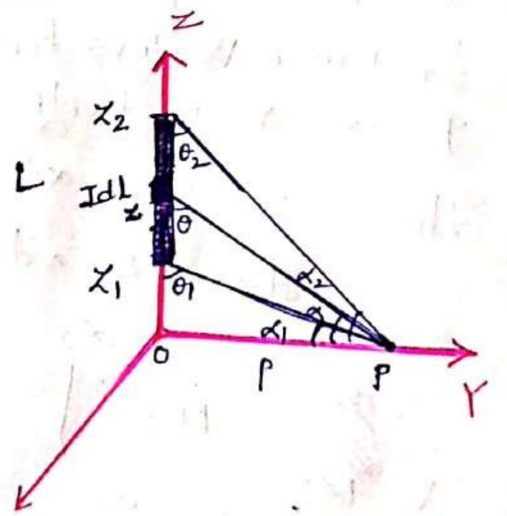
$L = 2\pi\rho$, the length of the magnetic path of radius ρ

$$\bar{H} = \frac{I}{L} \bar{a}_\phi = \frac{I\bar{a}_\phi}{2\pi\rho} \text{ A/m}$$

The magnetic flux density is

$$\boxed{\bar{B} = \mu \bar{H} = \frac{\mu I \bar{a}_\phi}{2\pi\rho} \text{ A/m}^2}$$

Magnetic field Intensity due to Finite length Conductor



Consider a straight Conductor of line finite length L carrying current I along the z-axis. Also consider a Current element $I\bar{dL}$ on the Conductor at a distance z from the origin. The end points of the Conductor are at distances z_1 and z_2 on the z-axis as shown in figure. Let a point P be on the y-axis at a distance ρ from the origin. Using cylindrical Coordinates, the position vector at the current element is $(0,0,z)$ and the position vector at point P is $(\rho,0,0)$

$\bar{R} = (\rho,0,0) - (0,0,z) = \rho\bar{a_\rho} - z\bar{a_z}$

$\bar{a_R} = \frac{\bar{R}}{|\bar{R}|} = \frac{\bar{R}}{R} = \frac{\rho\bar{a_\rho} - z\bar{a_z}}{\sqrt{\rho^2+z^2}}$

The current element is $Id\bar{L} = Idz\bar{a_z}$.

The magneticfield intensity at point p is

$$\bar{H} = \int \frac{Id\bar{L}\times\bar{a_R}}{4\pi R^2} = \int \frac{Id\bar{L}\times\bar{R}}{4\pi R^3}$$

$$Id\bar{L}\times\bar{R} = Idz\,\bar{a_z}\times\frac{(\rho\bar{a_\rho} - z\bar{a_z})}{\sqrt{\rho^2+z^2}} = \frac{Idz}{\sqrt{\rho^2+z^2}}\left(\rho\bar{a_z}\times\bar{a_\rho} - z\bar{a_z}\times\bar{a_z}\right)$$

$\bar{a_z}\times\bar{a_\rho} = \bar{a_\phi}$

$$Id\bar{L}\times\bar{R} = \frac{Idz}{\sqrt{\rho^2+z^2}}\rho\bar{a_\phi} \quad \Rightarrow \bar{H} = \int_{z_1}^{z_2}\frac{I\rho\bar{a_\phi}\,dz}{4\pi(\rho^2+z^2)^{3/2}}$$

To evaluate the line integral, let α, α_1 and α_2 be angles at point p in the clockwise direction that the distance vectors from points z, z_1 and z_2 makes with the y-axis respectively.

The $z = \rho\tan\alpha$ $\qquad z_1 = \rho\tan\alpha_1$ or $\alpha_1 = \tan^{-1}\left(\frac{z_1}{\rho}\right)$

$dz = \rho\sec^2\alpha\,d\alpha \qquad z_2 = \rho\tan\alpha_2$ or $\alpha_2 = \tan^{-1}\left(\frac{z_2}{\rho}\right)$

$$\bar{H} = \int_{\alpha_1}^{\alpha_2}\frac{I\rho\bar{a_\phi}\cdot\rho\sec^2\alpha\,d\alpha}{4\pi(\rho^2+\tan^2\alpha\,\rho^2)^{3/2}} = \int_{\alpha_1}^{\alpha_2}\frac{I\bar{a_\phi}}{4\pi\rho}\cos\alpha\,d\alpha$$

$$\bar{H} = \frac{I\bar{a_\phi}}{4\pi\rho}\left[\sin\alpha\right]_{\alpha_1}^{\alpha_2} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\bar{a_\phi}\ A/m$$

$$\bar{B} = \mu\bar{H} = \frac{\mu I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\bar{a_\phi}\ A/m$$

Scanned by CamScanner

Note: 1. If θ, θ_1 and θ_2 are the angles of the Conductor at points z, z_1 and z_2 to the point P from the z-axis as show in Figure. then $\theta = 90 - \alpha$, $\theta_1 = 90 - \alpha_1$, $\theta_2 = 90 - \alpha_2$

$$\text{So, } H = \frac{I}{4\pi\rho}\left[\sin(90-\theta_2) - \sin(90-\theta_1)\right]\bar{a}_\phi$$

$$= \frac{I}{4\pi\rho}(\cos\theta_2 - \cos\theta_1)\bar{a}_\phi$$

2. For infinitely long Conductors $\theta_2 = 0$ and $\theta_1 = 180°$

$$\bar{H} = \frac{I}{4\pi\rho}(\cos 0 - \cos 180)\bar{a}_\phi$$

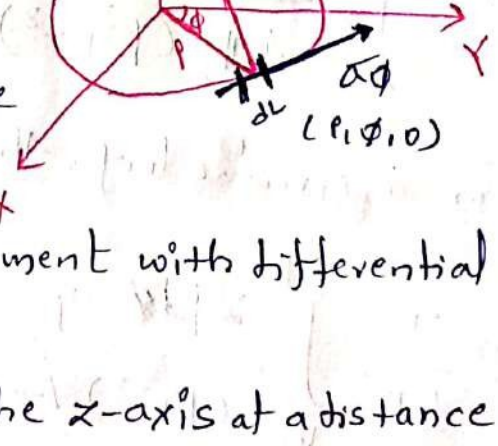
$$\bar{H} = \frac{I}{2\pi\rho}\bar{a}_\phi$$

3. For Semi-infinitely long Conductors $z_1 = 0$ and $z_2 = \infty$

$\theta_2 = 0°$ and $\theta_1 = 90°$

$$\text{then } \bar{H} = \frac{I}{4\pi\rho}(\cos 0 - \cos 90°) = \frac{I}{4\pi\rho}\bar{a}_\phi$$

Magnetic field Intensity along the axis of a circular loop:



Consider a Circular Loop Conductor of radius ρ on the x-Y plane centred at the origin as shown in figure Let the conductor carry a current I. The current element with differential length $d\bar{L}$ is $Id\bar{L}$.

Let P be a point on the z-axis at a distance 'z' from the origin.

Scanned by CamScanner

In cylindrical coordinates, the position vector at the current element is $(\rho, \phi, 0)$ and the position vector at point p is $(0, \phi, z)$

The distance vector is $\bar{R} = -\rho \bar{a_\rho} + z\bar{a_z}$

The unit vector is $\bar{a_R} = \dfrac{-\rho \bar{a_\rho} + z\bar{a_z}}{\sqrt{\rho^2+z^2}}$

The current element is $I d\bar{L} = I \rho d\phi \bar{a_\phi}$

The magnetic field intensity at point P is $\bar{H}$

$$\bar{H} = \int_L \frac{I d\bar{L} \times \bar{a_R}}{4\pi R^2}$$

Evaluating the cross product

$$I d\bar{L} \times \bar{a_R} = I \rho d\phi \bar{a_\phi} \times \frac{(-\rho \bar{a_\rho} + z\bar{a_z})}{\sqrt{\rho^2+z^2}}$$

$$= \frac{I}{\sqrt{\rho^2+z^2}} \rho d\phi \bar{a_\phi} \times (-\rho \bar{a_\rho} + z \bar{a_z})$$

$$\boxed{\begin{aligned} \bar{a_\rho} \times \bar{a_\phi} &= \bar{a_z} \\ \bar{a_\phi} \times \bar{a_z} &= \bar{a_\rho} \\ \bar{a_z} \times \bar{a_\rho} &= \bar{a_\phi} \end{aligned}}$$

$$= \frac{I}{\sqrt{\rho^2+z^2}} \left[\rho^2 d\phi \bar{a_z} + z\rho d\phi \bar{a_\rho}\right]$$

$$\bar{H} = \int \frac{I\left[z\rho d\phi \bar{a_\rho} + \rho^2 d\phi \bar{a_z}\right]}{4\pi (\rho^2+z^2)^{3/2}}$$

For a circular loop, the limits are from 0 to 2π.

$$\bar{H} = \frac{I}{4\pi} \int_0^{2\pi} \frac{z\rho \bar{a_\rho}}{(\rho^2+z^2)^{3/2}} d\phi + \frac{I}{4\pi} \int_0^{2\pi} \frac{\rho^2 \bar{a_z}}{(\rho^2+z^2)^{3/2}} d\phi$$

since the loop is symmetrical along the radial direction,
the $\bar{a}_\rho$ Component is zero

$$\bar{H} = \frac{I}{4\pi}\int_0^{2\pi} \frac{\rho^2 \bar{a}_z}{(\rho^2+z^2)^{3/2}}\, d\phi = \frac{I}{4\pi} \cdot \frac{\rho^2 \bar{a}_z}{(\rho^2+z^2)^{3/2}} \cdot 2\pi$$

$$\boxed{\bar{H} = \frac{I\rho^2}{2(\rho^2+z^2)^{3/2}}\,\bar{a}_z \text{ A/m}}$$

Thus the field exists only along the z-direction. In general, the field intensity is perpendicular to the plane of the Circular wire

Note1: If $z=0$, the magnetic field at the centre of the loop is

$$\boxed{\bar{H} = \frac{I\rho^2}{2(\rho^2+0)^{3/2}} = \frac{I}{2\rho}\,\bar{a}_z \text{ A/m}}$$

$$\boxed{\bar{B} = \mu\bar{H} = \frac{\mu I}{2\rho}\,\bar{a}_z \text{ wb/m}^2}$$

Note 2: If ~~the~~ L is the length of the circular loop ie $L = 2\pi\rho$
then the field at the centre of the loop is $\rho = \frac{L}{2\pi}$

$$\boxed{\bar{H} = \frac{I\,\bar{a}_z}{2\frac{L}{2\pi}} = \frac{\pi I}{L}\,\bar{a}_z \text{ A/m}}$$

$$\boxed{\bar{B} = \mu\bar{H} = \frac{\mu\pi I}{L}\,\bar{a}_z \text{ wb/m}^2}$$

Note 3: At a large distances from the loop, where $z >> \rho$
$\rho^2+z^2 \approx z^2$. The magnetic field is then given by

$$\boxed{\bar{H} = \frac{I\rho^2}{2z^3}\,\bar{a}_z \text{ A/m}}$$

$$\boxed{\bar{B} = \mu\bar{H} = \frac{\mu I\rho^2}{2z^3}\,\bar{a}_z \text{ wb/m}^2}$$

Problem: A current of 10A is flowing in the $\bar{a}_z$ direction on the z-axis. Find the field intensity $\bar{H}$ at point $P(1,2,3)$ due to this filament if it extends from $z=-\infty$ to $z=\infty$

Solution:

$$\bar{H} = \frac{I}{2\pi\rho}\bar{a}_\phi \qquad \rho = \sqrt{x^2+y^2} = \sqrt{1+4} = \sqrt{5}$$

$$\boxed{\bar{H} = \frac{10}{2\pi\sqrt{5}}\bar{a}_\phi = 0.7117\,\bar{a}_\phi \text{ A/m}}$$

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

$\rho = \sqrt{x^2+y^2}$, $\phi = \tan^{-1}(y/x)$, $z = z$

$$a_\rho = \cos\phi\,\bar{a}_x + \sin\phi\,\bar{a}_y$$
$$\bar{a}_\phi = -\sin\phi\,\bar{a}_x + \cos\phi\,\bar{a}_y$$
$$a_z = a_z$$

$$\sin\phi = \frac{y}{\rho} = \frac{2}{\sqrt{5}}$$
$$\cos\phi = \frac{x}{\rho} = \frac{1}{\sqrt{5}}$$

$$\bar{a}_\phi = -\frac{2}{\sqrt{5}}\bar{a}_x + \frac{1}{\sqrt{5}}\bar{a}_y$$

$$\bar{H} = +0.7117\left[-\frac{2}{\sqrt{5}}\bar{a}_x + \frac{1}{\sqrt{5}}\bar{a}_y\right]$$

$$\boxed{\bar{H} = -0.6365\bar{a}_x + 0.3182\bar{a}_y \quad \text{A/m}}$$

b) $z=0$ to $5m$.

$$\bar{H} = \frac{I}{4\pi\rho}\bar{a}_\phi \qquad \rho = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$$

$$= \frac{10}{4\pi(\sqrt{5})}\bar{a}_\phi =$$

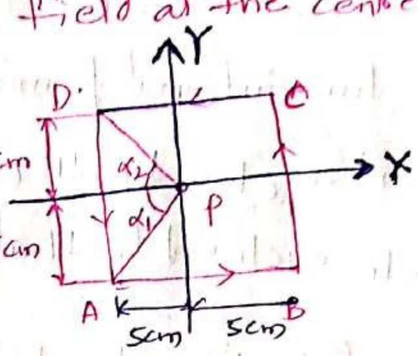
Scanned by CamScanner

Problem:

A wire carrying a current of 100 A is bent into a square of 10 cm side. Calculate the field at the centre of the square.

Solution:

Consider DA segment, which is finite length of the wire



$\alpha_1 = -\tan^{-1}(5/5) = -45^\circ$

$\alpha_2 = \tan^{-1}(5/5) = 45^\circ$

$$\bar{H} = \frac{I}{4\pi r}\left(\sin\alpha_2 - \sin\alpha_1\right) = \frac{100}{4\pi \times 5 \times 10^{-2}}\left[\sin 45^\circ - (-\sin 45)\right]$$

$= 225.079$ A/m

As the square in X-Y plane, the direction of $\bar{H}$ is normal to the X-Y plane i.e. $\bar{a_z}$

$\bar{H} = 225.079\,\bar{a_z}$ A/m

Such 4 Sides Contribute $\bar{H}$ at the Centre of the square at p hence $= 4 \times \bar{H} = 4 \times 225.0794 \bar{a_z} = 900.3163$ A/m

Problem: A circular loop located on $x^2+y^2=9$, $z=0$ carries a direct current of 10 A along a_ϕ. Determine $\bar{H}$ at $(0,0,4)$ and $(0,0,-4)$.

Dec - 2017

Solution:

$$\bar{H} = \frac{I\rho^2}{2(\rho^2+z^2)^{3/2}}\bar{a_z} \text{ A/m}$$

$\bar{H}$ at $(0,0,4)$

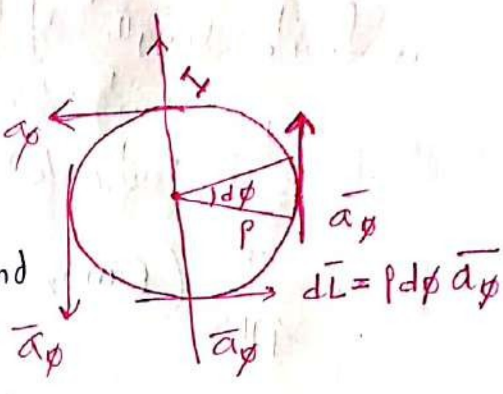
$I = 10$, $\rho = 3$, $z = 4$

$$= \frac{10 \times 9}{2(9+16)^{3/2}}\bar{a_z} = 0.36 A_z \text{ A/m}$$

$$\bar{H} \text{ at } (0,0,-4) = \frac{I\rho^2}{2(\rho^2+z^2)^{3/2}}\bar{a_z} = \frac{10 \times 9}{2(9+16)^{3/2}} = 0.36 A_z \text{ A/m}$$

Ampere's Circuital law or Ampere's work law

The line integral of magnetic field intensity $\bar{H}$ around a closed path is equal to the net current I_{enclo} enclosed by that path



$$\oint \bar{H} \cdot d\bar{L} = I_{encl}$$

where I_{encl} is the current enclosed by the path called Amperian path and $d\bar{L}$ the differential length.

Proof: Consider a long Conductor carrying current I along the x-axis as shown in figure. Take a closed circular path of radius ρ, called an Amperian path, around the Conductor. and current element $Id\bar{L}$

Using Biot-savart's law, $\bar{H}$ at a point p due to an infinite length current is

$$\bar{H} = \frac{I}{2\pi\rho}\bar{a}_\phi \qquad d\bar{L} = \rho d\phi\, \bar{a}_\phi$$

$$\bar{H} \cdot d\bar{L} = \frac{I}{2\pi\rho}\bar{a}_\phi \cdot \rho d\phi\, \bar{a}_\phi = \frac{I}{2\pi} d\phi$$

$$\oint \bar{H} \cdot d\bar{L} = \int_0^{2\pi} \frac{I}{2\pi} d\phi = \frac{I}{2\pi}[\phi]_0^{2\pi} = \frac{I}{2\pi} \times 2\pi = I$$

$I = I_{enclo}$, the current enclosed by the path.

Note 1: Ampere's Circuital law is analogous to Gauss's law in electrostatics. It can be applied to determine the magnetic field when the current distribution is symmetrical. The closed path (Amperian path) Considered need not be circular. It may be of any shape.

Note 2: The closed path on which the Ampere's circuital law is to be applied is called Amperian path. It is analogous to the Gaussian surface for electric fields. Generally, a Gaussian surface can be used for both electric and magnetic fields.

Note 3: For a Conductor cross-section $S\,m^2$ with current density J A/m^2, Ampere's law can be expressed as $I_{encl} = \oint \bar{H} \cdot d\bar{l} = \int_S \bar{J} \cdot d\bar{s}$

Applications of Ampere's law:

Ampere's law is useful to find the "$\bar{H}$" for some symmetrical current distributions like

i) An infinite line current
ii) An infinite current sheet
iii) An infinitely long coaxial transmission line.

Maxwell's equations:

From Ampere's Circuital law.

$$\int \bar{H} \cdot d\bar{l} = I_{enclosed}$$

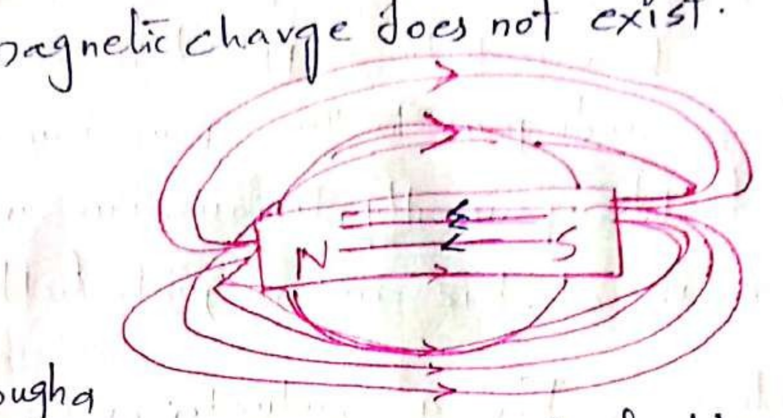
Ampere's law is a special case of Biot-savart's law

$$\int (\nabla \times \bar{H}) \cdot ds = \int \bar{J} \cdot d\bar{s}$$

$$\boxed{\nabla \times \bar{H} = \bar{J} \neq 0}$$

Magneto static field is not conservative

An isolated magnetic charge does not exist.



The total flux through a closed surface in a magnetic field must be zero.

$$\phi = \oint \bar{B} \cdot d\bar{s} = 0$$

$$\oint \bar{B} \cdot d\bar{s} = \int (\nabla \cdot \bar{B}) dv = 0$$

$$\boxed{\nabla \cdot \bar{B} = 0}$$

Maxwell's equations for static EM fields:

Differential (or point) form	Integral form	Remarks
1) $\nabla \cdot \bar{D} = \rho_v$	$\oint \bar{D} \cdot d\bar{s} = \int_V \rho_v dv$	Gauss's law
2) $\nabla \cdot \bar{B} = 0$	$\oint \bar{B} \cdot d\bar{s} = 0$	Nonexistence of magnetic monopole
3) $\nabla \times \bar{E} = 0$	$\oint_L \bar{E} \cdot d\bar{l} = 0$	Conservativeness of electrostatic field
4) $\nabla \times \bar{H} = \bar{J}$	$\oint_L \bar{H} \cdot d\bar{l} = \int \bar{J} \cdot ds$	Ampere's law.

Problem: In a certain conducting region $\bar{H} = yz(x^2+y^2)\bar{a_x} - y^2xz\bar{a_y} + 4x^2y^2\bar{a_z}$ mA/m

Determine $\bar{J}$ at $(5,2,-3)$

Solution:

$$\bar{H} = yz(x^2+y^2)\bar{a_x} - y^2xz\bar{a_y} + 4x^2y^2\bar{a_z} \text{ mA/m}$$

$$\nabla\times\bar{H} = \bar{J}$$

$$\nabla\times\bar{H} = \begin{vmatrix} \bar{a_x} & \bar{a_y} & \bar{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz(x^2+y^2) & -y^2xz & 4x^2y^2 \end{vmatrix}$$

$$= \bar{a_x}\left[\frac{\partial}{\partial y}(4x^2y^2) - \frac{\partial}{\partial z}(-y^2xz)\right] - \bar{a_y}\left[\frac{\partial}{\partial x}(4x^2y^2) - \frac{\partial}{\partial z}(yz(x^2+y^2))\right] + \bar{a_z}\left[\frac{\partial}{\partial x}(-y^2xz) - \frac{\partial}{\partial y}(yz(x^2+y^2))\right]$$

$$= \left[[8x^2y + y^2x]\bar{a_x} - [8xy^2 - x^2y - y^3]\bar{a_y} + [-y^2z - x^2z - 3y^2z]\bar{a_z}\right]\times 10^{-3}$$

substitute $x=5, y=2, z=-3$

$$= \left[[(8\times 5^2\times 2 + 2^2\cdot 5)]\bar{a_x} - [8\times 5\times 2^2 - 5^2\times 2 - (2)^3]\bar{a_y} + [-(2)^2(-3) - (5)^2(-3) - 3(2)^2(-3)]\bar{a_z}\right]\times 10^{-3}$$

$$= [420\bar{a_x} - 102\bar{a_y} + 123\bar{a_z}]\times 10^{-3}$$

$$= 0.420\bar{a_x} - 0.102\bar{a_y} + 0.123\bar{a_z} \text{ A/m}^2$$

Problem: A circular loop located at $x^2+y^2=9, z=0$ carries a direct current of 10 A along the $\bar{a_\phi}$ direction. Determine $\bar{H}$ at $(0,0,5)$ and $(0,0,-5)$

Solution:

$$\bar{H} = \frac{I\rho^2}{2(\rho^2+z^2)^{3/2}}\bar{a_z} \qquad \rho = \sqrt{9} = 3, \quad I = 10\text{ A}$$

i) At $(0,0,5)$: $\bar{H} = \frac{10(3)^2}{2(3^2+5^2)^{3/2}}\bar{a_z} = 0.226\,\bar{a_z}$ A/m

ii) At $(0,0,-5)$: $\bar{H} = \frac{10(3)^2}{2(3^2+5^2)^{3/2}}\bar{a_z} = 0.226\,\bar{a_z}$ A/m

Problem: The magnetic field intensity $\bar{H}$ due to current source is given by $\bar{H} = y\cos(ax)\,\bar{a_x} + ye^x\,\bar{a_z}$. Determine the current density over the y-z plane.

Solution:
From the maxwell's equations

$$\nabla \times \bar{H} = \bar{J}$$

$$\nabla \times \bar{H} = \begin{vmatrix} \bar{a_x} & \bar{a_y} & \bar{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y\cos(ax) & 0 & ye^x \end{vmatrix}$$

$$\nabla \times \bar{H} = \bar{a_x}\left[\frac{\partial}{\partial y}(ye^x) - \frac{\partial}{\partial z}(0)\right] - \bar{a_y}\left[\frac{\partial}{\partial x}(ye^x) - \frac{\partial}{\partial z}(y\cos ax)\right]$$
$$+ \bar{a_z}\left[\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial y}(y\cos ax)\right]$$

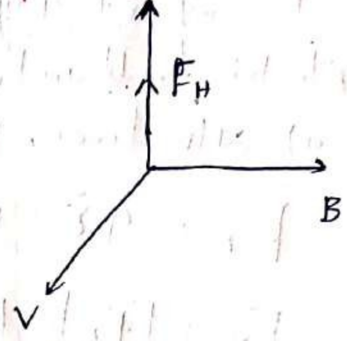
$$= \bar{a_x}\left[e^x - 0\right] - \bar{a_y}\left[ye^x - 0\right] + \bar{a_z}\left[0 - \cos ax\right]$$

$$= e^x\,\bar{a_x} - ye^x\,\bar{a_y} - \cos ax\,\bar{a_z}$$

On the y-z plane, $x = 0$

$$\bar{J} = \nabla \times \bar{H} = \bar{a_x} - y\bar{a_y} - \bar{a_z}.$$

Force on a moving charge Due to Electric field and Magnetic fields: (For Lorentz force equation)



If there is a charge or a moving Charge, Q in an electric field, E, there exists force on the charge

This force is given by $\overline{F_E} = EQ = Q\overline{E}$

If a charge, Q moving with a velocity, V is placed in an magnetic field B(μH), then there exists a force on the charge. This force is given by $\overline{F_H} = Q(\overline{V} \times \overline{B})$

If the charge, Q is placed in both electric and magnetic fields, then the force on the charge is

$$\overline{F} = Q\overline{E} + Q(\overline{V} \times \overline{B}) = Q(\overline{E} + \overline{V} \times \overline{B})$$

Q - Charge (Coulombs)
V - Velocity of Charge m/s
B - Magnetic fluxdensity (wb/m^2) or (Tesla)
E - Electric field Intensity (N/c or V/m)

This equation is known as Lorentz force equation.

Applications of Lorentz force equation:

1. Electron orbits in magnetrons
2. proton paths in cyclotrons.
3. plasma Charactestics in Magneto hydro Dynamic Generator (MHDG)
4. Motional Charactestics of a Charged body in combined electric and magnetic fields.

Problem: A charge of 12 c has velocity of $5a_x + 2a_y - 3a_z$ m/s. Determine F on the charge in the field of a) $\bar{E} = 18a_x + 5a_y + 10a_z$ V/m
b) $B = 4a_x + 4a_y + 3a_z$ wb/m^2 c) Find the total force due to Electric field and magnetic field.

Solution: a) The force, F_E on the charge, Q due to E is

$$\bar{F}_E = Q\bar{E} = 12(18a_x + 5a_y + 10a_z) = 216a_x + 60a_y + 120a_z$$

$$F_E = |\bar{F}_E| = |Q\bar{E}| = \sqrt{(216)^2 + (60)^2 + (120)^2} = 254.27\ N$$

b) The force F_H on the charge, Q due to B is

$$\bar{F}_H = Q(\bar{V} \times \bar{B}) = 12[18\bar{a}_x - 27\bar{a}_y + 12\bar{a}_z]$$

$$= 216\bar{a}_x - 324\bar{a}_y + 144\bar{a}_z$$

$$\bar{V} \times \bar{B} = \begin{vmatrix} a_x & a_y & a_z \\ 5 & 2 & -3 \\ 4 & 4 & 3 \end{vmatrix} = 18a_x - 27a_y + 12a_z$$

$$F_H = |\bar{F}_H| = \sqrt{(216)^2 + (-324)^2 + (144)^2}$$

$$= 415.17\ N$$

c) F (total force due to Electric field and magnetic field)

$$\bar{F} = \bar{F}_E + \bar{F}_Q$$

$$\bar{F} = 216\bar{a}_x + 60\bar{a}_y + 120\bar{a}_z + 216\bar{a}_x - 324\bar{a}_y + 144\bar{a}_z$$

$$\bar{F} = 432\bar{a}_x - 264\bar{a}_y + 264\bar{a}_z$$

$$F = |\bar{F}| = \sqrt{(432)^2 + (-264)^2 + (264)^2} = \text{[illegible]}\ N$$

$$\bar{F} = 432\bar{a}_x - 264\bar{a}_y + 264\bar{a}_z$$

Problem: An electron has velocity of 1 km/s along a_x in a magnetic field whose magnetic flux density is

$$\bar{B} = 0.2a_x - 0.3a_y + 0.5a_z\ \text{wb/m}^2.$$

a) Determine the electric field intensity if no force is applied ~~[illegible]~~ to the electron

b) also find the force on the electron under the influence of both E and B when $E = (a_x + a_y + a_z)$ kV/m

Solution: a) If $F = 0$ $\cancel{F = Q(E \times V}$

$$\bar{F} = Q(\bar{E} + \bar{V} \times \bar{B}) = 0$$

$$\bar{E} = -V \times B = -\begin{vmatrix} a_x & a_y & a_z \\ 10^3 & 0 & 0 \\ 0.2 & -0.3 & 0.5 \end{vmatrix}$$

$$\bar{E} = -\left[\bar{a}_x(0-0) - \bar{a}_y(10^3 \times 0.5 - 0) + \bar{a}_z(10^3 \times 0.3 - 0)\right]$$

$$\bar{E} = (0.5 a_y + 0.3 a_z) \text{ KV/m}$$

b) The force on the electron due to E and B is

$$\bar{F} = Q(\bar{E} + \bar{V} \times \bar{B})$$

$$Q = -1.6 \times 10^{-19} C$$

$$\bar{F} = -1.6 \times 10^{-19}\left[\bar{a}_x + \bar{a}_y + \bar{a}_z + 0.5\bar{a}_y + 0.3\bar{a}_z\right] \times 10^3$$

$$= (-1.6\bar{a}_x - 2.4\bar{a}_y - 2.08) \times 10^{-16} N$$

Force on a Current element in a magnetic field:

The force on a current element when placed in a magnetic field, B is

$$\bar{F} = I\bar{L} \times \bar{B}$$

$$F = ILB\sin\theta \text{ Newton}$$

'θ' is the angle between the direction of the current element and the direction of magnetic flux density

B – Magnetic flux density wb/m^2

IL – Current element Amp-m

Proof: Consider a differential charge, dq to be moving with a velocity, V in a magnetic field $H = B/\mu$. Then the differential force on the charge is given by

$$dF = dq\,(V \times B) \qquad dq = \rho_v\, dv$$

$$dF = \rho_v\, dv\,(V \times B)$$

$$= \rho_v\,(V \times B)\, dv$$

$$= (\rho_v V \times B)\, dv$$

$$\boxed{\rho_v V = \frac{C}{m^3}\cdot\frac{m}{s} = \frac{A}{m^2 \cdot s} = J}$$

$$d\bar{F} = J\,dv \times B$$

$$\boxed{I\,dL = K\,ds = J\,dv}$$

$$dF = I\,dL \times B$$

$$\boxed{F = \oint_S I\,dL \times B = \int_S K\,ds \times B = \int_v J\,dv \times B}$$

$$\bar{F} = I\bar{L} \times \bar{B},\ \text{Newton}$$

Problem: A current element 4 cm along y-axis with a current of 10mA flowing in y-direction. Determine the force on the current element due to the magnetic field if the magnetic field $H = \frac{5a_x}{\mu}$ A/m

Solution $\bar{F} = I\bar{L} \times \bar{B}$

$$IL = 10 \times 10^{-3} \times 4 \times 10^{-2}\,\bar{a}_y$$

$$= 4 \times 10^{-4}\,\bar{a}_y$$

$$\bar{B} = \mu H = \mu \cdot \frac{5a_x}{\mu} = 5a_x \ \text{wb/m}^2$$

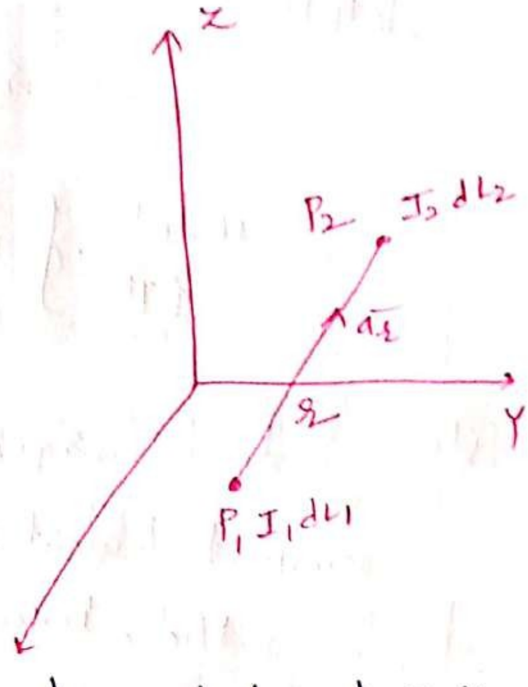
$$\bar{F} = I\bar{L} \times \bar{B} = 4 \times 10^{-4}\,\bar{a}_y \times 5\,\bar{a}_x$$

$$= -20 \times 10^{-4}\,\bar{a}_z = -2\,\bar{a}_z \ \text{mN}$$

Scanned by CamScanner

Ampere's force law:

Ampere's force law states that there exists a force between two current elements $I_1 dL_1$ and $I_2 dL_2$ and it's is given by



$$F = \frac{\mu I_1 I_2}{4\pi} \iint \frac{dL_2 \times (dL_1 \times a_r)}{r^2}$$

Proof: Consider two differential current elements $I_1 dL_1$ and $I_2 dL_2$ are separated by a distance 'r'

Let the differential current element $I_1 dL_1$ be at a point P_1 and $I_2 dL_2$ be at a point P_2.

The magnetic field at P_2 due to $I_1 dL_1$ is given by

$$dH = \frac{I_1 dL_1 \times a_r}{4\pi r^2}$$

$$dB = \frac{\mu I_1 dL_1 \times a_r}{4\pi r^2}$$

This field exerts a force on the current element $I_2 dL_2$ at point P_2 and it is given by

$$d(dF) = I_2 dL_2 \times dB$$

This is the differential of a differential force on a differential current element due to a differential field dB

$$d(dF) = I_2 dL_2 \times \frac{\mu I_1 dL_1 \times a_r}{4\pi r^2}$$

$$d(dF) = \frac{\mu I_1 I_2 \, dL_2 \times (dL_1 \times a_r)}{r^2}$$

$$\boxed{F = \frac{\mu I_1 I_2}{4\pi} \oint \oint \frac{dL_2 \times (dL_1 \times a_r)}{r^2} \text{ Newtons}}$$

Problem: In a magnetic flux density of $B = a_x + 3a_y \ wb/m^2$ a current element $10 a_x$ mA-m is placed. Find the force on the current element.

$$\bar{F} = I\bar{L} \times \bar{B}$$

$$I\bar{L} = 10\bar{a}_z \times 10^{-3} \text{ A-m}$$

$$\bar{B} = \bar{a}_x + 3\bar{a}_y \ wb/m^2$$

$$F = (10 \times 10^{-3} \bar{a}_z) \times (\bar{a}_x + 3\bar{a}_y)$$

$$= (-30\bar{a}_x + 10\bar{a}_y) \times 10^{-3} \text{ Newton}$$

Magnetic Scalar and Vector potentials:

Like scalar electrostatic potential, it is possible to have scalar magnetic potential. It is defined in such a way that its negative gradient gives the magnetic field that is.

$$\boxed{H = -\nabla V_m}$$

V_m = scalar magnetic potential (Amp)

Taking curl on both sides, we get

$$\boxed{\nabla \times H = -\nabla \times \nabla V_m}$$

But curl of the gradient of any scalar is always zero $\boxed{\nabla \times H = 0}$

But by Ampere's Circuital law $\nabla \times H = J$

$$J = 0$$

In other words, scalar magnetic potential exists in a region where $J=0$

The scalar potential satisfies Laplace's equation that is, we have

$$\nabla \cdot B = 0 \qquad B = \mu H$$

$$\nabla \cdot \mu H = 0$$

$$\nabla \cdot \mu(-\nabla V_m) = 0$$

$$-\mu \nabla^2 V_m = 0$$

$$\boxed{\nabla^2 V_m = 0 \quad \text{where } J = 0}$$

Laplace equation for scalar magnetic potential.

Scanned by CamScanner

Characteristics of Scalar magnetic potential (V_m)

1. The negative gradient of V_m gives H
2. It exists where $J=0$
3. It satisfies Laplace's equation
4. It is directly defined as $V_m = -\int_A^B \bar{H}\cdot d\bar{L}$
5. It has the unit of Ampere.

Problem: The magnetic field in a current free region $H = \frac{1}{\rho} a_\phi$. The region is defined by $1 \le \rho \le 2m$, $0 \le \phi \le 2\pi$ and $0 \le z \le 2m$. Find the scalar magnetic potential at $(4, 50°, 2)$.

Solution:

$$V_m = -\int \bar{H}\cdot d\bar{L}$$

$$H = \frac{1}{\rho}\bar{a}_\phi$$

$$d\bar{L} = d\rho\,\bar{a}_\rho + \rho\, d\phi\,\bar{a}_\phi + dz\,\bar{a}_z$$

$$V_m = -\int \frac{1}{\rho}\bar{a}_\phi \cdot (d\rho\,\bar{a}_\rho + \rho\, d\phi\,\bar{a}_\phi + dz\,\bar{a}_z)$$

$$= -\int \frac{1}{\rho}\cdot \rho\, d\phi = -\int d\phi = -\phi$$

$$V_m \text{ at } (4, 50°, 2) = -50 \times \pi/180 = -0.8726 \text{ Amp}$$

Vector magnetic potential :

Vector magnetic potential exists in regions where J is present. It is defined in such a way that ~~th~~ its curl gives the magnetic flux density

that is $\boxed{B = \nabla \times A}$

where A = Vector magnetic potential (wb/m)

$$\boxed{A = \int_L \frac{\mu_0 I dL}{4\pi R} \text{ for (line current)}}$$

$A = \int \mu \cancel{K}$

$$\boxed{A = \int_S \frac{\mu K ds}{4\pi R} \text{ for surface current}}$$

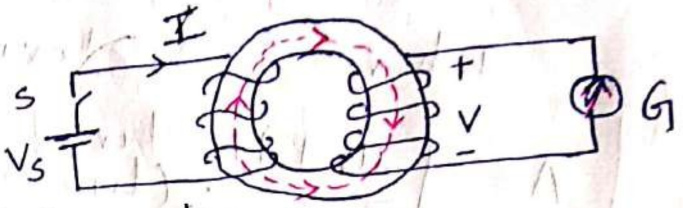
$$\boxed{A = \int \frac{\mu J dv}{4\pi R} \text{ for volume current}}$$

Charactustics of Vector magnetic potential:

1. It exists even when J is present
2. $\nabla^2 A = -\mu_0 J$
3. $\nabla^2 A = 0$ if $J = 0$
4. Vector magnetic potential, "A" has applications to obtain radiation characteristics of antennas, apertures and also to obtain radiation leakage from transmission lines, wave guides and microwave ovens.
5. "A" is used to find near and far fields of antennas.

Maxwell's equations (Time varying fields):

Faraday's law:



Faraday's law states that in a magnetic field, if a closed circuit is in motion or if the magnetic field is time-varying, then the induced electromagnetic force (emf) or voltage across the terminals of the closed circuit is equal to the time rate of magnetic flux linkage by the circuit

$$V = -N\frac{d\phi}{dt}$$

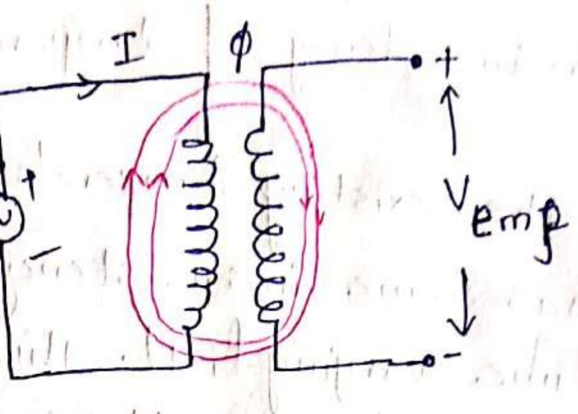
N - Number of turns of the circuit

ϕ - flux through each turn

The negative sign is explained by Lenz's law given below

Lenz's law: Lenz's law states that the direction of the induced emf is always such that it tends to set up an induced current separately, which produces a flux opposing the flux linkages. In other words, the direction of current flow in the closed circuit is such that the induced magnetic field produced by the current will oppose the original magnetic field.

Transformer emf:



Consider a transformer as shown in figure. When a time-varying current is applied, it produces a time-varying magnetic flux in the primary coil that induces an emf in the secondary coil. This emf is called transformer emf or statically induced emf.

According to faraday' law for $(N=1)$

$$V_{emf} = \frac{-d\phi}{dt}$$

we know that $\phi = \int \bar{B}\cdot d\bar{s}$

$$V_{emf} = \int \bar{E}\cdot d\bar{L}$$

$$\int \bar{E}\cdot d\bar{L} = -\frac{d}{dt}\left[\int \bar{B}\cdot d\bar{s}\right] = -\int \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$

using Stoke's theorem

$$\oint \bar{E}\cdot d\bar{L} = \oint (\nabla\times\bar{E})\cdot ds = -\int \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$

Eliminating the surface integrals $\boxed{\bar{B} = \mu\bar{H}}$

$$\boxed{\nabla\times\bar{E} = \frac{-\partial\bar{B}}{\partial t} = -\frac{\mu\,\partial\bar{H}}{\partial t}}$$

Maxwell's equations.

Time varying field is non consevative.

Modified Ampere's law and Displacement current Density (Inconsistency of Ampere's law):

The existing Ampere's law $\nabla \times \bar{H} = \bar{J}_c$, the law has some inconsistency or unsatisfaction for time varying field. This inconsistency is been eliminated by adding new term $\bar{J}_d$

The Ampere's law can be rewritten as

$$\boxed{\nabla \times \bar{H} = \bar{J}_c + \bar{J}_d}$$

(Applying divergence on both sides)

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot (\bar{J}_c + \bar{J}_d) = \nabla \cdot \bar{J}_c + \nabla \cdot \bar{J}_d$$

Divergence of a curl is zero

$$\cancel{\nabla} \cdot 0 = \nabla \cdot \bar{J}_c + \nabla \cdot \bar{J}_d \Rightarrow \nabla \cdot \bar{J}_d = -\nabla \cdot \bar{J}_c$$

By Continuity equation $\boxed{\nabla \cdot J_c = \frac{-\partial \rho_v}{\partial t}}$ = $\boxed{\begin{array}{c}\nabla \cdot \bar{D} = \rho_v \\ \text{Gauss law}\end{array}}$

$$\nabla \cdot \bar{J}_c = \frac{-\partial}{\partial t}(\nabla \cdot \bar{D}) = -\nabla \cdot \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot \bar{J}_d = -\left(-\nabla \cdot \frac{\partial \bar{D}}{\partial t}\right) = \nabla \cdot \frac{\partial \bar{D}}{\partial t}$$

Suppress Divergence on both sides

$$\boxed{\cancel{\nabla} \cdot \bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \frac{\varepsilon \partial \bar{E}}{\partial t}} \qquad \bar{D} = \varepsilon \bar{E}$$

$\bar{J}_d$: Displacement current density (A/m^2)

$\bar{J}_d$ is defined as the time rate of change of electric flux density.

$\bar{J}_c$ dominates in a conducting medium and is zero in a perfect dielectric

$\bar{J}_d$ dominates in a dielectric medium and is zero in a perfect conductor.

Ratio between Conduction Current density ($\bar{J}_c$) and Displacement Current Density ($\bar{J}_d$)

Let $\bar{E}$ be a time-varying field given by

$$\bar{E} = E e^{j\omega t}$$

The Conduction Current density is $\bar{J}_c = \sigma \bar{E}$

and the displacement current density $\bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \varepsilon \frac{\partial \bar{E}}{\partial t}$

$$\bar{J}_d = \varepsilon \frac{\partial \bar{E}}{\partial t} = \varepsilon \frac{\partial}{\partial t}(E e^{j\omega t}) = j\omega \epsilon \bar{E}$$

Ratio between $\bar{J}_c$ and $\bar{J}_d$ is

$$\frac{\bar{J}_c}{\bar{J}_d} = \frac{\sigma \bar{E}}{j\omega \epsilon \bar{E}} \Rightarrow \left|\frac{\bar{J}_c}{\bar{J}_d}\right| = \frac{\sigma}{\omega \epsilon}$$

At low frequencies, the displacement current density J_d is usually negligible compared to conduction current density $\bar{J}_c$. But at ratio frequencies (high frequencies), in wave propagation, the value of J_d becomes comparable with J_c.

Note: The current densities depend on the medium constants σ, ε and frequency ω ($2\pi f$). The medium is classified based on the current densities in the wave to..

1. if $\frac{\sigma}{\omega\epsilon} >> 1$, the medium is a perfect Conductor
2. if $\frac{\sigma}{\omega\epsilon} << 1$, the medium is a perfect dielectric
3. if $\frac{\sigma}{\omega\epsilon} = 0$, the medium is an insulator or freespace

Scanned by CamScanner

Differences between displacement current density and Conduction Current density:

Conduction current density (J_c)	Displacement current density (J_d)
1) It is defined as the Conduction current at a given point, passing through a unit surface area normal to the direction of the current. It is denoted by $\bar{J_c}$	1) It is defined as the displacement current at a given point, passing through a unit surface area when the surface is normal to the direction of the displacement current. It is denoted by $\bar{J_d}$
2) Conduction current results in Conductors according to ohm's law	2) Displacement current results when a potential is applied across the dielectric medium.
3) It obey's ohm's law	3) It does not obeys ohm's law
4) Conduction Current density is given by $J_c = \sigma \bar{E}$	4) Displacement current density is given by $\bar{J_d} = \frac{\partial \bar{D}}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t}$
5) only Conduction current exists and displacement current density is negligible in Conductors	5) only displacement current density exists and Conduction current density is negligible in a dielectric medium
6) Conduction ~~Displacement~~ current density exists ~~only in time~~- both in Steady and time-varying fields	6) Displacement current density exists only in time-varying fields
7) Examples: Current flowing through Conductors and Resistors	7) Examples: current flowing through Capacitor and all imperfect Conductors

Differences between Conduction, Convection and Displacement Current

Conduction current	Convection current	Displacement current
Conduction current is the flow of electrons through any conducting medium	Convection current is the flow of electrons a non-conducting (insulating) medium	Displacement current is the flow of charge in dielectrics which results due to a time-varying electric fields.
Conduction current is the current passing through resistors and conductors	It is the leakage current passing through the dielectric (insulating) medium of the capacitor.	It is the rate of flow of charge between the capacitor plates in a capacitor circuit.
It is independent of frequency	Its value increases with frequency	It is directly proportional to frequency
It obeys ohm's law and hence has linear charge characteristics	It does not obey ohm's law and so has non-linear characteristics	It also does not obey ohm's law and so has non-linear characteristics
It exists in both time variant and invariant cases	It also exists in both time variant and invariant cases	It exists only in time variant case.
6. It flows only through closed paths in circuits	It flows only through open paths in circuits	It provides a closed path in circuits having capacitor elements or in open circuits where the conduction current cannot flow further
7. Examples: Current through conductors, resistor	Electron beam moving through vaccum, CRT, liquids	Current flowing through capacitor and all imperfect conductors carrying a time-varying conduction current

Maxwell's equations

1) The first equation states that the magnetomotive force (mmf) around a closed path is equal to the sum of the conduction current and displacement current through any surface bounded by the path (modified ~~Ampere's~~ Circuital law)

$$\boxed{\nabla \times \bar{H} = \bar{J}_c + \bar{J}_d = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t}}$$ point form or Differential form

$$\boxed{\int \bar{H} \cdot d\bar{L} = \int \left(\bar{J}_c + \frac{\partial \bar{D}}{\partial t}\right) d\bar{s}}$$ Integral form

2) The second equation states that the electromotive force (emf) around a closed path is equal to the negative of the time derivative of the magnetic flux density through any surface bounded by the path (Faraday's law of Induction).

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}} \qquad \boxed{\int \bar{E} \cdot d\bar{L} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}}$$

3) The third equation states that the total electric flux density through any surface enclosing a volume is equal to the total charge within that volume. (Gauss's law)

$$\boxed{\nabla \cdot \bar{D} = \rho_v} \qquad \boxed{\int \bar{D} \cdot d\bar{s} = \int \rho_v \, dv}$$

4) The fourth equation states that the total magnetic flux emerging through any closed surface is zero.

(Gauss' law for magnetic fields or Non existence of single magnetic pole)

$$\boxed{\nabla \cdot \bar{B} = 0} \qquad \boxed{\oint \bar{B} \cdot d\bar{s} = 0}$$

5) The Continuity equation ~~is states that the total~~

$$\boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} = -\dot{\rho}_v} \qquad \boxed{\int \bar{J} \cdot d\bar{s} = -\int \frac{\partial \rho_v}{\partial t} dv}$$

Maxwell's equations for free space and time varying fields

$(\sigma = 0,\ J = 0,\ \rho_v = 0)$

Point (Differential) form	Integral form
1) $\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$	$\oint \bar{H} \cdot d\bar{l} = \int \frac{\partial \bar{D}}{\partial t} \cdot d\bar{s}$
2) $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint \bar{E} \cdot d\bar{l} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$
3) $\nabla \cdot \bar{D} = 0$	$\oint \bar{D} \cdot d\bar{s} = 0$
4) $\nabla \cdot \bar{B} = 0$	$\oint \bar{B} \cdot d\bar{s} = 0$

Maxwell's equations for free space and static fields

$(\sigma = 0)$

Point (Differential) form	Integral form
1) $\nabla \times \bar{H} = 0$	$\oint \bar{H} \cdot d\bar{l} = 0$
2) $\nabla \times \bar{E} = 0$	$\oint \bar{E} \cdot d\bar{l} = 0$
3) $\nabla \cdot \bar{D} = 0$	$\oint \bar{D} \cdot d\bar{s} = 0$
4) $\nabla \cdot \bar{B} = 0$	$\oint \bar{B} \cdot d\bar{s} = 0$

Maxwell's equations for a good conductor $(\sigma >> \omega\epsilon,\ \rho_v = 0)$

Point (Differential form)	Integral form
1) $\nabla \times \bar{H} = \bar{J}$	$\oint \bar{H} \cdot d\bar{l} = \int \bar{J} \cdot d\bar{s} = I$
2) $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint \bar{E} \cdot d\bar{l} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$
3) $\nabla \cdot \bar{D} = 0$	$\oint \bar{D} \cdot d\bar{s} = 0$
4) $\nabla \cdot \bar{B} = 0$	$\oint \cdot \bar{B} \cdot d\bar{s} = 0$

Boundary Conditions on E, D, H, B

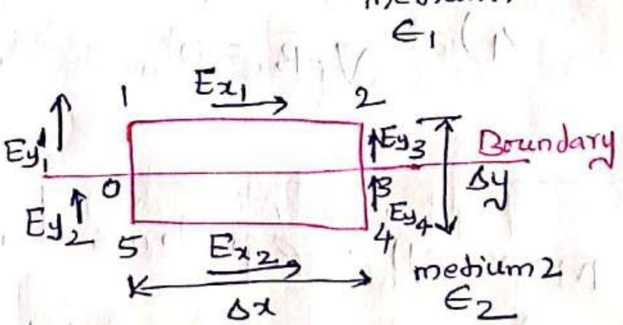
a) Boundary Conditions for Electric field Intensity (E)

when a field crosses any media interface, it causes an abrupt change in its media properties such as ε, μ, and σ. The electric and magnetic field components across the surface boundaries vary due to the discontinuities in the properties of material. The integral form of maxwell's equation when applied at the media interface gives relation between the field components of media. These relations are termed as 'Boundary Conditions'.

In other words, boundary conditions specify the field values at the surface of the boundary

a) Boundary Conditions for Electric field Intensity ($\bar{E}$):

Consider the rectangular loop on the boundary of two media.



It is well-known that electric field is conservative and hence the line integral of $\bar{E}\cdot d\bar{L}$ is zero around a closed path.

$$\oint \bar{E}\cdot d\bar{L} = 0$$

From the figure shown above LHS is written as

$$LHS = \int_{01} + \int_{12} + \int_{23} + \int_{34} + \int_{45} + \int_{50}$$

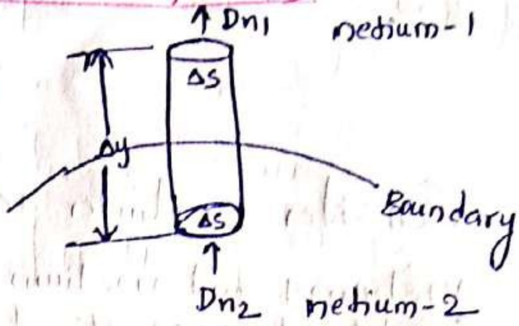
$$= E_{y_1}\frac{\Delta y}{2} + E_{x_1}\Delta x - E_{y_3}\frac{\Delta y}{2} - E_{y4}\frac{\Delta y}{2} - E_{x2}\Delta x + E_{y2}\frac{\Delta y}{2} = $$

As $\Delta y \to 0$ $\Rightarrow E_{x_1}\Delta x - E_{x2}\Delta x = 0 \Rightarrow E_{x_1} = E_{x2}$

E_{x_1}, E_{x_2} are the tangential components of E in medium 1 and 2 respectively

$$\boxed{E_{tan1} = E_{tan2}}$$

b) Boundary Conditions for Electric flux density ($\bar{D}$)



Consider a cylinder across the medium-1 and medium-2

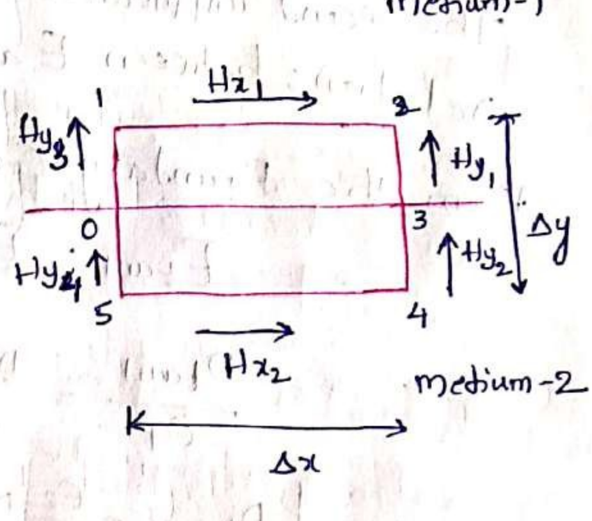
According to Gauss law $\int \bar{D}\cdot d\bar{s} = Q$

Applying this to the cylindrical surface on the boundary sprating over medium 1 and medium 2, we get as $\Delta y \to 0$

$$\cancel{D_{n_1}\Delta S}\cdot \; D_{n_1}\Delta S - D_{n_2}\Delta S = Q$$

$$\boxed{D_{n1} - D_{n2} = \frac{Q}{\Delta S} = \rho_s}$$

c) Boundary Conditions for magnetic field intensity ($\bar{H}$):



Consider the rectangular loop on the boundary of two media.

From the Ampere's law, we have

$$\oint \bar{H}\cdot d\bar{l} = \int_{01} + \int_{12} + \int_{23} + \int_{34} + \int_{45} + \int_{50} = I$$

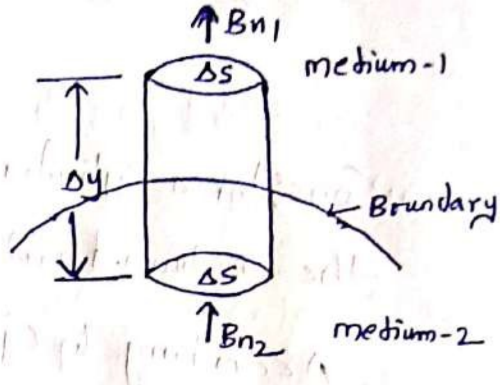
$$= H_{y_3}\frac{\Delta y}{2} + H_{x_1}\Delta x - H_{y_1}\frac{\Delta y}{2} - H_{y_2}\frac{\Delta y}{2} - H_{x_2}\Delta x + H_{y_4}\frac{\Delta y}{2} = I$$

As $\Delta y \to 0$, we get

$$\Rightarrow H_{x_1}\Delta x - H_{x_2}\Delta x = I \Rightarrow H_{x_1} - H_{x_2} = \frac{I}{\Delta x} = J_s$$

Here H_{x_1}, H_{x_2} are nothing but tangential components in medium-1 and medium-2 respectively $\boxed{H_{tan1} - H_{tan2} = J_s}$

d) Boundary Conditions for magnetic flux density ($\bar{B}$):



Consider a cylinder across medium-1 and medium-2.

Gauss's law for magnetic fields is

$$\int \bar{B} \cdot d\bar{s} = 0$$

$$B_{n1} \Delta S - B_{n2} \Delta S = 0$$

$$\boxed{B_{n1} = B_{n2}}$$

Given the fields in one medium, it is usually required to determine fields in a second medium. This requires the knowledge of both tangential and Normal components for each field. The above boundary conditions yield either tangential or normal components. The second unknown component is determined from the constitutive relations between E and D, H and B.

From the Boundary Conditions of $\bar{E}$

$$E_{tan1} = E_{tan2}$$

$$D = \epsilon E$$

$$\frac{D_{tan1}}{\epsilon_1} = \frac{D_{tan2}}{\epsilon_2}$$

$$\boxed{D_{tan2} = \frac{\epsilon_2}{\epsilon_1} D_{tan1} = \frac{\epsilon_{r2}}{\epsilon_{r1}} D_{tan1}}$$

From the Boundary Conditions of $\bar{D}$

$$D_{n1} - D_{n2} = \rho_s$$

$$\boxed{\epsilon_1 E_{n1} - \epsilon_2 E_{n2} = \rho_s}$$

Scanned by CamScanner

From the Boundary Conditions of $\bar{B}$

$$B_{n1} = B_{n2}$$

$$B = \mu H$$

$$\mu_1 H_{n1} = \mu_2 H_{n2}$$

$$\boxed{H_{n1} = \frac{\mu_2}{\mu_1} H_{n2} = \frac{\mu_{r2}}{\mu_{r1}} H_{n2}}$$

From the Boundary Conditions of $\bar{H}$

$$H_{tan1} - H_{tan2} = J_s$$

$$\frac{B_{tan1}}{\mu_1} - \frac{B_{tan2}}{\mu_2} = J_s$$

$$\boxed{\frac{B_{tan1}}{\mu_{r1}} - \frac{B_{tan2}}{\mu_{r2}} = \mu_0 J_s}$$

Boundary Conditions at the Interface between Two Dielectrics:

As a dielectric medium has zero Conductivity (σ), Current density (J_s) also becomes zero.

$$\boxed{J = J_s}$$

$$\left.\begin{array}{l} \sigma = 0 \Rightarrow \bar{J} = \sigma \bar{E} = 0 \\ \rho_s = 0 \end{array}\right\} \text{in medium-1 \& medium-2}$$

$\boxed{a)\ E_{tan1} = E_{tan2}}$

$\boxed{b)\ D_{n1} - D_{n2} = \rho_s \Rightarrow D_{n1} - D_{n2} = 0 \Rightarrow D_{n1} = D_{n2}}$

$\boxed{c)\ H_{tan1} - H_{tan2} = J_s \Rightarrow H_{tan1} - H_{tan2} = 0 \Rightarrow H_{tan1} = H_{tan2}}$

$\boxed{d)\ B_{n1} = B_{n2}}$

Boundary Conditions at the Interface of Dielectric and Perfect Conductor:

medium-1 is dielectric & medium-2 is Conductor

For Conductor the Conductivity is infinite

The Electric field ($\bar{E}$) and magnetic field ($\bar{H}$) inside a Conductor is zero.

$E_{tan2} = 0, \; H_{tan2} = 0, \; D_{n2} = 0, \; B_{n2} = 0$

a) $\boxed{E_{tan1} = E_{tan2} \Rightarrow E_{tan1} = 0}$

b) $\boxed{D_{n1} - D_{n2} = \rho_s \Rightarrow D_{n1} = \rho_s}$

c) $\boxed{H_{tan1} - H_{tan2} = J_s \Rightarrow H_{tan1} = J_s}$

d) $\boxed{B_{n1} = B_{n2} \Rightarrow B_{n1} = 0}$

Inductance:

Inductor: It is a coil of wire wound according to various designs with or without a core of magnetic material to concentrate the magnetic field.

Inductance of a conductor system is defined as the ratio of magnetic flux linkage to the current producing the flux, the is $\boxed{L = \frac{N\phi}{I}}$ Henry

N = Number of turns
ϕ = flux produced
I = current in coil.

or ability to store the enery in the form of magnetic field. Units are Henry or Web/Amp

$\boxed{\text{Energy stored in an inductor is } W_L = \frac{1}{2} L I^2}$

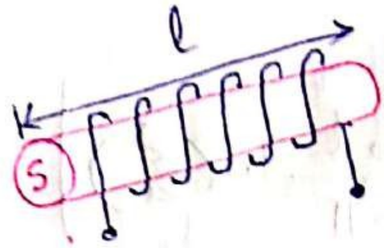
$\boxed{\text{Energy density in an inductor is } W_H = \frac{1}{2}\mu H^2 = \frac{1}{2} BH}$

Standard Inductance Configurations:

a) Solenoid:

Inductance

$$\boxed{L = \frac{\mu_0 N^2 S}{l}}$$



N = Number of turns

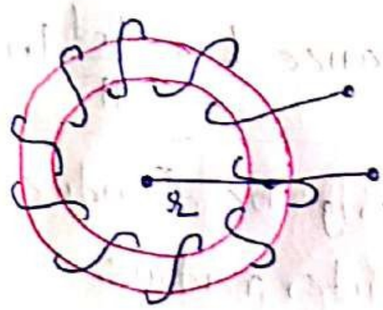
S = Cross-Sectional Area

l = length of solenoid

b) Toroid:

Inductance

$$\boxed{L = \frac{\mu_0 N^2 S}{2\pi r}}$$



N = Number of turns

r = Average radius

S = Cross-Sectional Area.

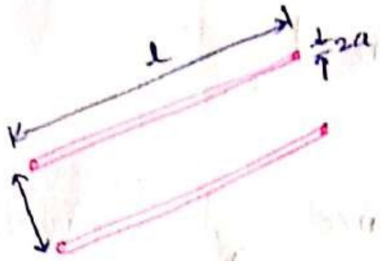
c) Wire:

$$\boxed{L = \frac{\mu_0 l}{8\pi}}$$

l = length of the wire

a = radius of the wire

4) Inductance of parallel wires:



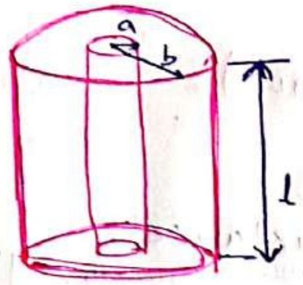
Inductance

$$\boxed{L = \frac{\mu_0 l}{\pi} \ln\left(\frac{d}{a}\right)}$$

$l >> d$
$d >> a$

l - length of the wire
d - distance between the wires
a - Radius of wire

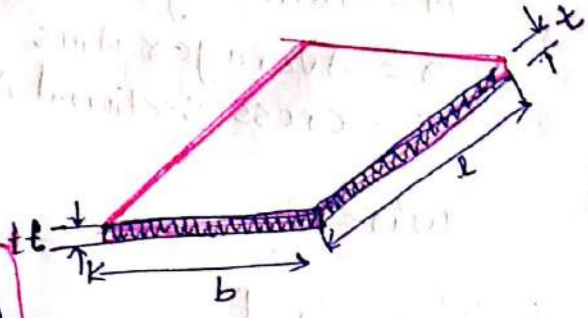
5) Coaxial Conductor:



$$\boxed{\text{Inductance } L = \frac{\mu_0 l}{\pi} \ln\left(\frac{b}{a}\right)}$$

l - leng the Conductor
b - outer radius
a - inner radius

6) Sheet:



Inductance

$$\boxed{L = \mu_0 2l\left(\ln\left(\frac{2l}{b+t}\right) + 0.5\right)}$$

l = length of sheet
b = breadth of sheet
t = thickness of sheet

specifications of Inductors:

1) Inductance value
2) Type of Core (material)
3) Type of winding (single, multi layer)
4) Frequency
5) Quality factor of the coil ($Q = \frac{X_L}{R}$)
6) Coupling factor
7) stability
8) Self capacitance.

Scanned by CamScanner