UNIT-I-Electrostatics:

Syllabus: Coulomb's law, Electric field Intensity, Fields due to different charge distributions, Electric flux density, Gauss law and applications, Electric potential, Relations between E and V, maxwell's Two equations for Electrostatic fields. Energy density, Illustrative problems. Convection and Conduction Currents, Dielectric Constants, Isotropic and Homogeneous Dielectrics, Continuity equation, Relaxation time. poisson's and laplace's equations, capacitance - parallel plate, coaxial, spherical Capacitors, Illustrative problems.

Introduction:

Electrostatic fields are also called static electric fields or steady electric fields. These fields are not variant with time. They are produced by static charges or charge distributions. These fields have a wide range of applications.

Applications of Electrostatic fields:

1. In cathode ray tube oscilloscopes to obtain electron beam deflection
2. In Ink-jet printers to obtain speed of printing and quality of print.
3. to Sort out minerals in ore separators, to sort out seeds in agriculture and for spraying plants and trees.
4. In Electrostatic generators
5. to produce potential
6. to produce force on charges for their mobility
7. In electric power transmission

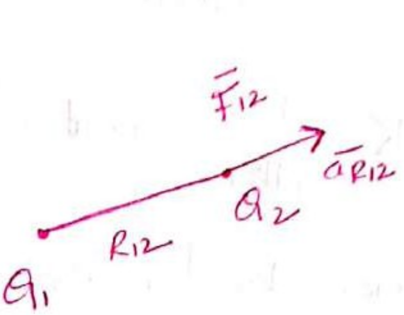
8. for lightning protection
9. to measure moisture content
10. to spin cotton
11. for field-field effect transistors (FET's)
12. for Capacitors
13. for LCD's
14. for touch pads
15. for capacitance keyboards
16. for fast baking of bread
17. for ECGs, EEGs, in medical applications
18. for spray painting
19. for electrodeposition
20. for electrochemical machining
21. X-Ray machines
22. Computer peripherals.

Coulomb's law: Charles Augustin De Coulomb (1736-1806) was a french physicist. After conducting several experiments on charged bodies, he concluded that there exists a force between them. He formulated a law known as coulomb's law.

Coulomb's law states that the force between two point charges Q_1 and Q_2, separated by a distance R_{12} is proportional to the product of the charges and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the charges. Coulomb also postulated that like charges repel and unlike charges attract eachother.

$$\overline{F}_{12} \propto Q_1 Q_2$$

$$\overline{F}_{12} \propto \frac{1}{R_{12}^2}$$



$$\overline{F}_{12} \propto \frac{Q_1 Q_2}{R_{12}^2}$$

$$\overline{F}_{12} = K \cdot \frac{Q_1 Q_2}{R_{12}^2}$$

Where Q_1 and Q_2 are positive or negative quantities of Charge in Coulombs (c) and R_{12} is the magnitude of the radial distance between the charges in metres (m), $\bar{a}_{R12}$ is the unit vector in the radial direction from Q_1 to Q_2 and K is the proportionality constant.

Coulomb'law is illustrated in above figure. The point charges located at a point in the medium are very small in size, negligible compared to the distance between them. Charge of an electron e^- is equal to 1.6×10^{-19} Coulombs.

The proportionality Constant K can be determined from practical experiments. It depends on the dielectric constant (permittivity) of the medium.

For Convenience, it can be expressed as

$$K = \frac{1}{4\pi\varepsilon}$$

where $\varepsilon = \varepsilon_0 \varepsilon_r$ is the dielectric constant or permittivity of the medium

ε_r = Relative permittivity of the medium (dimensionless quantity)

ε_0 = permittivity of vacuum (absolute permittivity) F/m

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$$\varepsilon_0 = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12}\ F/m$$

$$K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ m/F$$

Therefore the force between two charges Q_1 and Q_2 in free space is

$$\bar{F}_{12} = 9\times10^9 \times \frac{Q_1 Q_2}{R_{12}^2}\, \bar{a}_{R12}$$

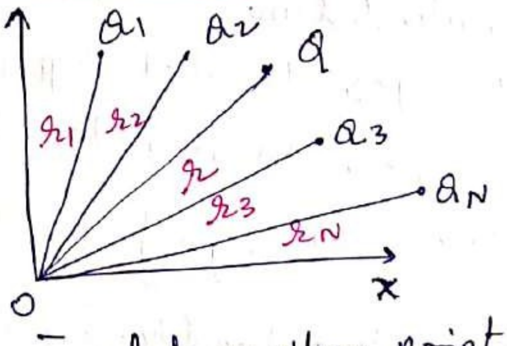
and magnitude of force is

$$F_{12} = |\bar{F}_{12}| = \frac{9\times10^9 \times Q_1 Q_2}{R_{12}^2}\ \text{Newtons}$$

ε_r - values of ε_r for materials

Material - ε_r
Vacuum - 1.0000
Air - 1.0006
Teflon - 2.10
polypropylene - 2.20-2.28
polysterence - 2.4-3.2
wood (oak) - 3.3
Bakelite - 3.5-6.0
wood (maple) - 4.4
Dry clay - 4.8
Glass - 4.9-7.5
wood (Birch) - 5.2
mica - 6.3-9.3
procelun - 6.5
wet clay - 6.32
ice - 3.5
water - 81

Forces due to N number of charges:



Consider N number of charges $Q_1, Q_2, \cdots Q_N$ as shown in figure with associated vectors $\bar{r}_1, \bar{r}_2, \cdots \bar{r}_N$. let another point charge Q be at position vector $\bar{r}$

The total force exterted on a point charge Q due to N Number of charges satisfies the principle of superposition i.e, it is equal to the vector sum of all the forces exerted on that charge due to each of the other point charges.

let the vectors $\bar{R}_1, \bar{R}_2, \cdots \bar{R}_N$ be the distance vectors from Q to $Q_1, Q_2 \cdots Q_N$ respectively. Then the forces $\bar{F}_1\ \bar{F}_2 \cdots \bar{F}_N$ exerted on Q due to $Q_1, Q_2 \cdots Q_N$ respectively are

$$\bar{F}_1 = \frac{Q Q_1}{4\pi\varepsilon R_1^2}\bar{a}_{R_1}$$

$$\bar{F}_2 = \frac{Q Q_2}{4\pi\varepsilon R_2^2}\bar{a}_{R_2}$$

$$\vdots$$

and $$\bar{F}_N = \frac{Q Q_N}{4\pi\varepsilon R_N^2}\bar{a}_{R_N}$$

The respective unit vectors are

$$\bar{a}_{R_1} = \frac{\bar{R}_1}{|\bar{R}_1|} = \frac{\bar{r}-\bar{r}_1}{|\bar{r}-\bar{r}_1|}$$

$$\bar{a}_{R_2} = \frac{\bar{R}_2}{|\bar{R}_2|} = \frac{\bar{r}-\bar{r}_2}{|\bar{r}-\bar{r}_2|}$$

and $$\bar{a}_{R_N} = \frac{\bar{R}_N}{|\bar{R}_N|} = \frac{\bar{r}-\bar{r}_N}{|\bar{r}-\bar{r}_N|}$$

Hence the total force on Q_1 according to the principle of superposition is

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \cdots \bar{F}_N$$

$$= \frac{Q Q_1}{4\pi\varepsilon R_1^2}\bar{a}_{R_1} + \frac{Q Q_2}{4\pi\varepsilon R_2^2}\bar{a}_{R_2} + \cdots + \frac{Q Q_N}{4\pi\varepsilon R_N^2}\bar{a}_{R_N}$$

$$\bar{F} = \frac{Q}{4\pi\varepsilon}\sum_{n=1}^{N}\frac{Q_n \bar{a}_{R_n}}{R_n^2} = \frac{Q}{4\pi\varepsilon}\sum_{n=1}^{N}\frac{Q_n \bar{R}_n}{(R_n)^3}$$

$$\bar{a}_{R_N} = \frac{\bar{R}_N}{|\bar{R}_N|}$$

$$R_N = |\bar{R}_N|$$

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Applications of Coulomb's law:

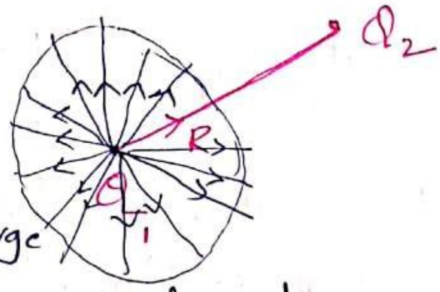
Coulomb's law is used to

1. Find the force between a pair of charges
2. find the potential at a point due to a fixed charge
3. find the electric field at a point due to a fixed charge
4. find displacement flux density indirectly
5. find the potential and electric field due to any type of charge distribution.
6. find the charge if the force and the electric field are known.

limitations of Coulomb's law:

It is difficult to apply the law when charges of are of arbitrary shape. Here the distance r cannot be determined accurately as the centres of arbitrarily shaped charged bodies cannot be identified accurately.

Electric field Intensity:



The force exerted on a unit charge in an electric field is called electric field intensity ($\bar{E}$) or Electric field strength. The electric field intensity at any point due to a point charge is proportional to the magnitude of the charge and inversely proportional to the square of the distance from the charge to that point.

If a point charge is brought near another charge, it experiences a force. The field where a charge exerts a force on any another charge is called the electric field of that charge.

Consider a single point charge Q_2 located at any point near another fixed point charge Q_1, as shown in figure. The electric field strength due to point charge Q_1 at the point charge Q_2 is defined as the force per unit charge at Q_2

$$\bar{F} = \frac{Q_1 Q_2}{4\pi\varepsilon R_{12}^2}\,\bar{a}_{R_{12}}$$

$$\bar{E} = \frac{\bar{F}}{Q_2} = \frac{Q_1}{4\pi\varepsilon R_{12}^2}\,\bar{a}_{R_{12}} \quad N/C \text{ or } V/m$$

$\bar{R}_{12} = \bar{r}_2 - \bar{r}_1$; is the radial distance between Q_1 and Q_2

Here, the fixed charge Q_1 becomes the source of the force acting on Q_2. The vector field $\bar{E}$ gives the force per unit charge anywhere in space.

In general, the electric field intensity is defined as

$$\bar{E} = \lim_{Q \to 0} \frac{\bar{F}}{Q} \quad N/C \text{ or } V/m$$

The vector field, the electric field intensity is proportional to the force and the direction is along the force.

It can be seen that the electric field intensity is symmetrical in the radial direction and varies inversely with square of the distance.

If a positive charge Q is located at the centre of an imaginary sphere of radius R with radial unit vector $\bar{a}_R$ directed radially outwards from the centre, the electric field intensity $\bar{E}$ at any point on the spherical surface is given by

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon R^2}\bar{a}_R \ \ V/m} \qquad \bar{a}_R = \frac{\bar{R}}{|R|} = \frac{\bar{R}}{R}$$

$$R = |\bar{R}|$$

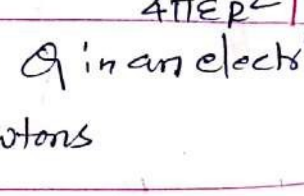
The direction of $\bar{E}$ is along the distance from the positive charge Q i.e. electric field has the same intensity at all points on the spherical surface.

The magnitude of Electric field strength is $E = \dfrac{Q}{4\pi\varepsilon R^2}$

Note: The force on a stationary charge Q in an electric field $\bar{E}$ is given by $\bar{F} = Q\bar{E}$ Newtons

Electric field Intensity due to N number of Charges:

Consider N charges $Q_1, Q_2, Q_3 \cdots Q_N$ located at different points defined by position vectors $\bar{r}_1, \bar{r}_2, \bar{r}_3 \cdots \bar{r}_N$ with unit vectors, $\bar{a}_{R_1}, \bar{a}_{R_2} \cdots \bar{a}_{R_N}$. Since the field is linear, the principle of superposition holds good. The total electric field intensity $\bar{E}$ at any point P at distance r from the origin is the vector sum of the individual electric field intensities caused by all the individual charges at point P. It is expressed as



$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \cdots + \bar{E}_N$$

$$\bar{E} = \frac{Q_1}{4\pi\varepsilon R_1^2}\bar{a}_{R_1} + \frac{Q_2}{4\pi\varepsilon R_2^2}\bar{a}_{R_2} + \cdots + \frac{Q_N}{4\pi\varepsilon R_N^2}\bar{a}_{R_N}$$

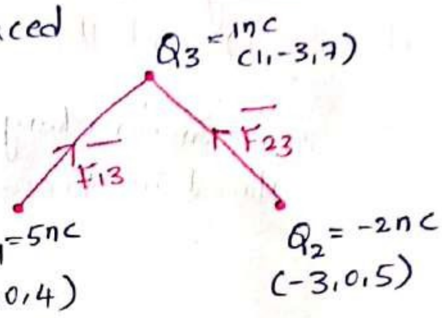
$$\bar{R}_n = \bar{r} - \bar{r}_n$$

$$\bar{E} = \frac{1}{4\pi\varepsilon}\sum_{n=1}^{N}\frac{Q_n}{R_n^2}\bar{a}_{R_n} \qquad \bar{a}_{R_n} = \frac{\bar{r} - \bar{r}_n}{|\bar{r} - \bar{r}_n|}$$

(1)

Point charges 5 nC and −2 nC are located at (2,0,4) and (−3,0,5) respectively. a) Determine the force on a 1-nC point charge located at (1,−3,7)
b) Find the electric field $\bar{E}$ at (1,−3,7) $\boxed{\text{Dec-2017}}$

Consider Charges placed in Free space



Total Force at charge Q_3 (1nC) is

$$\bar{F} = \bar{F}_{13} + \bar{F}_{23}$$

$$\bar{F}_{13} = \frac{Q_1 Q_3}{4\pi\varepsilon_0 (R_{13})^3}\bar{R}_{13} = 5\times10^{-9} \times (-2\times10^{-9}) \times 9\times10^{9}$$

$$\bar{R}_{13} = r_3 - r_1 = (1,-3,7) - (2,0,4) = (-1,-3,3)$$

$$= -\bar{a}_x - 3\bar{a}_y + 3\bar{a}_z$$

$$R_{13} = |\bar{R}_{13}| = \sqrt{(-1)^2 + (-3)^2 + (3)^2} = \sqrt{19}$$

$$\bar{F}_{13} = \frac{5\times10^{-9} \times (1\times10^{-9}) \times 9\times10^{9} \times (-\bar{a}_x - 3\bar{a}_y + 3\bar{a}_z)}{(\sqrt{19})^3} \quad \text{—— (1)}$$

$$= 0.543(-\bar{a}_x - 3\bar{a}_y + 3\bar{a}_z) \times 10^{-9} = \underline{(-0.543\bar{a}_x - 1.629\bar{a}_y + 1.629\bar{a}_z)\ nN}$$

$$\bar{F}_{23} = \frac{Q_1 Q_3}{4\pi\varepsilon_0 (R_{23})^3}\bar{R}_{23}$$

$$\bar{R}_{23} = r_3 - r_2 = (1,-3,7) - (-3,0,5) = (4,-3,2)$$

$$= 4\bar{a}_x - 3\bar{a}_y + 2\bar{a}_z$$

$$R_{23} = |\bar{R}_{23}| = \sqrt{4^2 + (-3)^2 + (2)^2} = \sqrt{16+9+4} = \sqrt{29}$$

$$\bar{F}_{23} = \frac{-2\times10^{-9} \times 1\times10^{-9} \times 9\times10^{9} (4\bar{a}_x - 3\bar{a}_y + 2\bar{a}_z)}{(\sqrt{29})^3} \quad \text{—— (2)}$$

$$= -0.115(4\bar{a}_x - 3\bar{a}_y + 2\bar{a}_z) \times 10^{-9} = \underline{(-0.46\bar{a}_x + 0.345\bar{a}_y - 0.23\bar{a}_z)\ nN}$$

$$\bar{F} = \bar{F}_{13} + \bar{F}_{23} = -1.003\bar{a}_x - 1.284\bar{a}_y + 1.4\bar{a}_z \ nN$$

Electric field Intensity ($\bar{E}$):

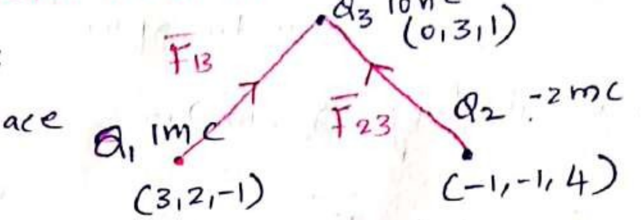
$$\bar{E} = \frac{\bar{F}}{Q} = \frac{-1.003\bar{a}_x - 1.284\bar{a}_y + 1.4\bar{a}_z \times 10^{-9}}{1\times10^{-9}}$$

$$= \underline{-1.003\bar{a}_x - 1.284\bar{a}_y + 1.4\bar{a}_z \ V/m \text{ or } N/C}$$

(2)

Point charges 1mC and −2mC are located at (3,2,−1) and (−1,−1,4) respectively. Calculate the electric force on a 10-nC charge located at (0,3,1) and the electric field intensity at that point.

Consider charges placed in Free space



Total Force at charge $\overset{Q_3}{\text{10-nC}}$ is $\bar{F} = \bar{F}_{13} + \bar{F}_{23}$

$$\bar{F}_{13} = \frac{Q_1 Q_3}{4\pi\varepsilon_0 R_{13}^2}\bar{a}_{R13} = \frac{Q_1 Q_3}{4\pi\varepsilon_0}\frac{\bar{R}_{13}}{(R_{13})^3}$$

$$\bar{R}_{13} = r_3 - r_1 = (0,3,1) - (3,2,-1) = (-3,1,2) = -3\bar{a}_x + \bar{a}_y + 2\bar{a}_z$$

$$R_{13} = |\bar{R}_{13}| = \sqrt{(-3)^2 + (1)^2 + (2)^2} = \sqrt{14}$$

$$\bar{F}_{13} = \frac{1\times10^{-3}\times10\times10^{-9}\times9\times10^{9}(-3\bar{a}_x + \bar{a}_y + 2\bar{a}_z)}{(\sqrt{14})^3} \quad \text{—— (1)}$$

$$\bar{F}_{23} = \frac{Q_2 Q_3}{4\pi\varepsilon_0}\frac{\bar{R}_{23}}{(R_{23})^3}$$

$$\bar{R}_{23} = r_3 - r_2 = (0,3,1) - (-1,-1,4) = (1,4,-3) = \bar{a}_x + 4\bar{a}_y - 3\bar{a}_z$$

$$R_{23} = |\bar{R}_{23}| = \sqrt{(1)^2 + (4)^2 + (-3)^2} = \sqrt{26}$$

$$\bar{F}_{23} = \frac{-2\times10^{-3}\times10\times10^{-9}\times9\times10^{9}(\bar{a}_x + 4\bar{a}_y - 3\bar{a}_z)}{(\sqrt{26})^3} \quad \text{—— (2)}$$

$$\bar{F} = \bar{F}_{13} + \bar{F}_{23} = -6.507\bar{a}_x - 3.817\bar{a}_y + 7.506\bar{a}_z \text{ mN}$$

Electric field Intensity (E):

$$\bar{E} = \frac{\bar{F}}{Q} = \frac{(-6.507\bar{a}_x - 3.817\bar{a}_y + 7.506\bar{a}_z)\times10^{-3}}{10\times10^{-9}}$$

$$= (-6.507\bar{a}_x - 3.817\bar{a}_y + 7.506\bar{a}_z)\times10^{5} \text{ V/m or N/C}$$

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Determine the total charge

a) on line $0 < x < 5m$ if $\rho_L = 12x^2$ mc/m

b) on the cylinder $\rho = 3$, $0 < z < 4$ m if $\rho_s = \rho z^2$ nc/m^2

c) Within the sphere $r = 4$ m if $\rho_v = \frac{10}{r\sin\theta}$ c/m^3

a) $$Q = \oint_L \rho_L dL = \int_{x=0}^{5} 10^{-3} \times 12x^2 dx = 12\left[\frac{x^3}{3}\right] \times 10^{-3} = 12\left[\frac{5^3}{3}\right] \times 10^{-3}$$

$$= \underline{500 \times 10^{-3} = 0.5\,C}$$

$1\bar{e} = 1.6 \times 10^{-19} C$

$1C = \frac{1}{1.6 \times 10^{-19}} \bar{e}$

$1C = 6.25 \times 10^{18} e^-$

$ds = \rho\, d\phi\, dz$

b) $$Q = \oint_s \rho_s ds = \int_{z=0}^{4} \int_{\phi=0}^{2\pi} \rho z^2 \cdot \rho\, d\phi\, dz \times 10^{-9}$$

$$= \rho^2 \int_{z=0}^{4} \int_{\phi=0}^{2\pi} z^2 d\phi\, dz \times 10^{-9} = 9 \cdot \left[\frac{z^3}{3}\right]_0^4 [\phi]_0^{2\pi} \times 10^{-9}$$

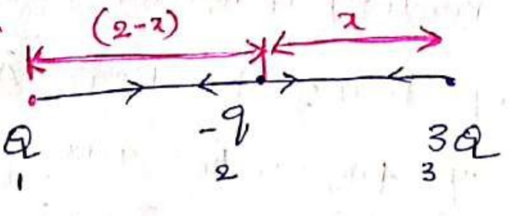
$$= 9 \times 2\pi \times \frac{64}{3} \times 10^{-9} = \underline{1.206\,\mu C}$$

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

c) $$Q = \oint \rho_v dv = \int_{r=0}^{4} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{10}{r\sin\theta} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 10\left[r^2/2\right]_0^4 [\phi]_0^{2\pi} [\theta]_0^{\pi} = \underline{1579.1\,C}$$

(4) Charges $+Q$ and $+3Q$ are separated by a distance 2m. A third charge is located such that the electrostatic system is in equilibrium. Find the location and the value of the third charge in terms of Q.



$$\overline{F_{12}} = \overline{F_{23}} = \overline{F_{13}}$$

$$\frac{Q(-q)}{4\pi\varepsilon_0(2-x)^2} = \frac{-3Qq}{4\pi\varepsilon_0 x^2} = \frac{3QQ}{4\pi\varepsilon_0(2)^2}$$

$3(2-x)^2 = x^2$

$3(4 - 4x + x^2) = x^2 \implies 2x^2 - 12x + 12 = 0$

$x^2 - 6x + 6 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(6)}}{2 \times 1}$$

$$x = \frac{6 \pm \sqrt{12}}{2} = 3 \pm \sqrt{3}$$

$$x = 3+\sqrt{3},\ 3-\sqrt{3}$$

$$= 4.732,\ \underline{1.2679}$$

Equate

$$\frac{-3qQ}{4\pi x^2} = \frac{3Q^2}{4\pi \cdot 4}$$

$$x = 1.2679$$

$$\frac{-\cancel{3}q\cancel{Q}}{\cancel{4\pi}(1.2679)^2} = \frac{\cancel{3}Q^2}{\cancel{4\pi}(4)}$$

$$-q = \frac{1.2679^2}{4} Q = 0.4018Q$$

$$\boxed{q = -0.4018Q\ c}$$

5) For a given vector $A = (x+y)\bar{a_x} + 3\bar{a_y} + x^2\bar{a_z}$. Determine the unit vector in the direction of $-A$ at the Cartesian coordinate point $(1,1,1)$. [April-2018]

$$A = (x+y)\bar{a_x} + 3\bar{a_y} + x^2\bar{a_z}$$

$$(x,y,z) = (1,1,1) \quad x=1, y=1, z=1$$

$$\bar{A} = (1+1)\bar{a_x} + 3\bar{a_y} + (1)^2\bar{a_z} = 2\bar{a_x} + 3\bar{a_y} + \bar{a_z}$$

$$-\bar{A} = -2\bar{a_x} - 3\bar{a_y} - \bar{a_z}$$

Unit vector in the direction of $-\bar{A}$ is $= \frac{-\bar{A}}{|-\bar{A}|}$

$$= \frac{-2\bar{a_x} - 3\bar{a_y} - \cancel{3}\bar{a_z}}{\sqrt{2^2+3^2+1}} = \frac{-2\bar{a_x} - 3\bar{a_y} - \bar{a_z}}{\sqrt{14}}$$

Given vectors $A = 3\bar{a_y} + 4\bar{a_z}$ and $B = 4\bar{a_x} - 10\bar{a_y} + 5\bar{a_z}$ determine

i) The vector component of A in the direction of B [April-2018]

ii) A unit vector perpendicular to both A and B.

The projection of A on B is $\bar{A} \cdot a_B = \frac{\bar{A} \cdot \bar{B}}{|\bar{B}|}$

$$\bar{A} = 3\bar{a_y} + 4\bar{a_z}, \quad \bar{B} = 4\bar{a_x} - 10\bar{a_y} + 5\bar{a_z}$$

$$\bar{a}_A = \frac{3\bar{a_y} + 4\bar{a_z}}{\sqrt{9+16}} = \frac{3\bar{a_y} + 4\bar{a_z}}{5}$$

$$\bar{a}_B = \frac{4\bar{a_x} - 10\bar{a_y} + 5\bar{a_z}}{\sqrt{16+100+25}} = \frac{4\bar{a_x} - 10\bar{a_y} + 5\bar{a_z}}{\sqrt{141}}$$

The projection of A on B is $\bar{A} \cdot \bar{a}_B = (3\bar{a_y} + 4\bar{a_z}) \cdot \frac{(4\bar{a_x} - 10\bar{a_y} + 5\bar{a_z})}{\sqrt{141}}$

$$= \frac{-\cancel{40}10}{\sqrt{141}}$$

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The vector Component of A in the direction of B

$$(\bar{A}\cdot \bar{a_B})\, a_B = \frac{\cancel{-140}\,-10}{\sqrt{141}}\frac{(4\bar{a_x}-10\bar{a_y}+5\bar{a_z})}{\sqrt{141}}$$

$$= \frac{\cancel{-160\bar{a_x}+400\bar{a_y}-200\bar{a_z}}}{141} = \frac{-40\bar{a_x}+100\bar{a_y}-50\bar{a_z}}{(141)}$$

$$= \cancel{-1.13\bar{a_x}+2.83\bar{a_y}-1.418\bar{a_z}} = \cancel{-8.36\bar{a_x}+8.42\bar{a_y}-4.2\bar{a_z}}$$

$$\boxed{-0.2831\bar{a_x}+0.7092\bar{a_y}-0.3546\bar{a_z}}$$

ii) A unit vector perpendicular to both $\bar{A}$ and $\bar{B}$

$$a_{AB} = \frac{\pm(\bar{A}\times\bar{B})}{|\bar{A}\times\bar{B}|} =$$

$$\bar{A}\times\bar{B} = \begin{vmatrix} \bar{a_x} & \bar{a_y} & \bar{a_z} \\ 0 & 3 & 4 \\ 4 & -10 & 5 \end{vmatrix} = \bar{a_x}\begin{vmatrix} 3 & 4 \\ -10 & 5 \end{vmatrix} - \bar{a_y}\begin{vmatrix} 0 & 4 \\ 4 & 5 \end{vmatrix} + \bar{a_z}\begin{vmatrix} 0 & 3 \\ 4 & -10 \end{vmatrix}$$

$$= 55\bar{a_x} - 16\bar{a_y} - 12\bar{a_z}$$

$$|\bar{A}\times\bar{B}| = \sqrt{(55)^2+(-16)^2+(-12)^2} = 58.523$$

$$a_{AB} = \pm\frac{(\bar{A}\times\bar{B})}{|\bar{A}\times\bar{B}|} = \pm\frac{(55\bar{a_x}-16\bar{a_y}-12\bar{a_z})}{58.523} = \pm(0.9398\bar{a_x}+0.2733\bar{a_y}-0.205\bar{a_z})$$

7) Derive the relation between electric field, E and Scalar potential. Find the electric field at (2,3,1), if the potential distribution is V of the form $3x^2y+y^2x+3z$. $(x,y,z)=(2,3,1)$ $\boxed{\text{May-2016}}$

$$\bar{E} = -\nabla V = \left(\frac{\partial}{\partial x}\bar{a_x}+\frac{\partial}{\partial y}\bar{a_y}+\frac{\partial}{\partial z}\bar{a_z}\right)(3x^2y+y^2x+3z)$$

$$= \frac{\partial}{\partial x}(3x^2y+y^2x+3z)\bar{a_x}+\frac{\partial}{\partial y}(3x^2y+y^2x+3z)\bar{a_y}+\frac{\partial}{\partial z}(3x^2y+y^2x+3z)\bar{a_z}$$

$$= (6xy+y^2)\bar{a_x}+(3x^2+2xy)\bar{a_y}+3\bar{a_z}$$

Substitute $x=2,\ y=3,\ z=1$

$$= (6\times2\times3+3^2)\bar{a_x}+(3\cdot2^2+2\cdot2\cdot3)\bar{a_y}+3\bar{a_z}$$

$$= 45\bar{a_x}+24\bar{a_y}+3\bar{a_z}\ \text{V/m}$$

Applications of Coulomb's law:

1. Find the force between a pair of charges
2. find the potential at a point due to a fixed charge
3. find the electric field at a point due to a fixed charge
4. find the displacement flux density indirectly
5. Find the potential and electric field due to any type of charge distribution.
6. find the charge if the force and and the electric field are known.

Salient features of electric Intensity:

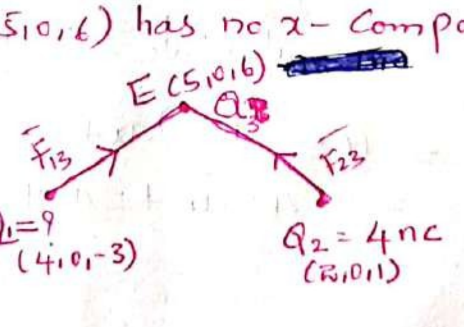
1. It has units of newton/coulomb or volts/metre
2. It is a vector
3. It has both direction and magnitude
4. It's direction is the same as that of Coulomb's law
5. It's magnitude depends on the magnitude of Coulomb's force and the charge on which the force is acting.
6. It depends on medium (permittivity)
7. It depend's on the distance of the charge from another charge which produces Coulomb's force
8. It depends on the location of charges.
9. It originates from a positive charge and terminates on a negative charge.

(8) point charges Q_1 and Q_2 are, repectively, located at $(4,0,-3)$ and $(2,0,1)$. if $Q_2 = 4nc$ find Q_1 such that

a) The E at $(5,0,6)$ has no x-component

b) The force on a test charge $(5,0,6)$ has no x-component.

Consider the charges placed in free space $\varepsilon = \varepsilon_0$



$$\bar{E} = \frac{Q_1}{4\pi\varepsilon_0}\frac{\bar{R_{13}}}{(R_{13})^3} + \frac{Q_2\,\bar{R_{23}}}{4\pi\varepsilon_0 (R_{23})^3}$$

$$E = Q_1 \times 9\times10^9 \frac{[(5,0,6)-(4,0,-3)]}{[|(5,0,6)-(4,0,-3)|]^3} + 4\times10^{-9}\times9\times10^9\frac{[(5,0,6)-(2,0,1)]}{[|(5,0,6)-(2,0,1)|]^3}$$

$$= \frac{Q_1 \times 9\times10^9\,(1,0,9)}{(\sqrt{82})^3} + \frac{4\times10^{-9}\times9\times10^9\;(3,0,5)}{(\sqrt{34})^3}$$

According to data z-component $=0$

$$= \frac{Q_1\times9\times10^9\,(9)}{(\sqrt{82})^3} + \frac{4\times10^{-9}\times9\times10^9\,(5)}{(\sqrt{34})^3} = 0$$

$$= Q_1 = \frac{-4\times10^{-9}\times9\times\cancel{10^9}\,(5)}{(\sqrt{34})^3}\times\frac{(\sqrt{82})^3}{9\times\cancel{10^9}\times9}$$

$$= -8.323\times10^{-9} = -8.323\,nC$$

$$\bar{F} = \bar{F_{13}} + \bar{F_{23}}$$

$$= \frac{Q_3\,Q_1(\bar{R_{13}})}{4\pi\varepsilon_0\,(R_{13})^3} + \frac{Q_3\,Q_2\,(\bar{R_{23}})}{4\pi\varepsilon_0\,(R_{23})^3}$$

$$= \frac{Q_3\times Q_1\times[(5,0,6)-(4,0,-3)]\times9\times10^9}{[|(5,0,6)-(4,0,-3)|]^3} + \frac{Q_3\times4\times10^{-9}\times9\times10^9[(5,0,6)-(2,0,1)]}{[|(5,0,6)-(2,0,1)|]^3}$$

$$= \frac{Q_3\times Q_1\times[1,0,9)\times9\times10^9}{(\sqrt{82})^3}\;\frac{Q_3\times4\times10^{-9}\times9\times10^9\,(3,0,5)}{(\sqrt{34})^3}$$

According to data x-component $=0$

$$\frac{\cancel{Q_3}\times Q_1\times\cancel{9\times10^9}\times(\cancel{1,0,9})}{(\sqrt{82})^3} + \frac{\cancel{Q_3}\times4\times10^{-9}\times\cancel{9\times10^9}\,(\cancel{3,0,5})\,3}{(\sqrt{34})^3}$$

$$\frac{Q_1}{(\sqrt{82})^3} + \frac{4\times10^{-9}\times3}{(\sqrt{34})^3} \Rightarrow Q_1 = \frac{-4\times10^{-9}\times3}{(\sqrt{34})^3}\times(\sqrt{82})^3 = -168.339\times10^{-9} = -168.339\,nC$$

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) Find the charge in the volume defined by $0 \le x \le 2m, 0 \le y \le 2m, 0 \le z \le 2m$.
if $\rho_v = 30y^2x \ \mu C/m^3$

$$Q = \oint_v \rho_v dv = \int_{x=0}^{2}\int_{y=0}^{2}\int_{z=0}^{2} 30y^2 x\, dx\,dy\,dz \times 10^{-6} = 30\left[\frac{x^2}{2}\right]\left[y^3/3\right]\left[z\right]_0^2 \times 10^{-6}$$

$$= 30 \times 2^2/2 \times 2^3/3 \times 2 \times 10^{-6} = \underline{320 \times 10^{-6} \mu C} = 320 \times 10^{-6} \times 6.25 \times 10^{18}$$
$$= 2 \times 10^{15} e^-$$

b) $\rho_v = 100 e^{-1000\rho} e^{-100z} \ C/m^3$ $\quad 0 \le \rho \le 0.01m$, $0 \le \phi \le 2\pi$, $0 \le z \le 0.01m$ $\quad dv = \rho\, d\rho\, d\phi\, dz$

$$Q = \int \rho_v dv = \int 100 e^{-1000\rho} e^{-100z} \cdot \rho\, d\rho\, d\phi\, dz = 100 \int_{\rho=0}^{0.01} \rho e^{-1000\rho} d\rho \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{0.01} e^{-1000z} dz$$

$$= 100 [\phi]_0^{2\pi} \left[\frac{e^{-1000z}}{-100}\right]_0^{0.01} \left[\rho \cdot \frac{e^{-1000\rho}}{-1000} - 1 \cdot \frac{e^{-1000\rho}}{-1000 \times -1000}\right]_0^{0.01}$$

$$= 100 [2\pi] \left[\frac{-0.3678}{100} + \frac{1}{100}\right] \left[\left[4.539 \times 10^{-10} - 4.539 \times 10^{-11}\right] - \left[0 - 10^{-6}\right]\right] = 100 \times 2\pi \times 6.322 \times 10^{-3} \times 1 \times 10^{-6}$$
$$= 3.970 \times 10^{-6}$$
$$= \underline{3.97 \mu C}$$

c) $\rho_v = 10^{-5} e^{-100r} \sin\theta \ C/m^3$ for $0 \le r \le 1\,cm$ $\quad dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$Q = \int_{r=0}^{0.01}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 10^{-5} e^{-100r} \sin\theta \cdot r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 10^{-5} \int_{r=0}^{0.01}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 e^{-100r} \sin^2\theta\, dr\, d\theta\, d\phi$$

$$= 10^{-5} [\phi]_0^{2\pi} \left[\theta - \sin\theta\cos\theta\right]_0^{\pi} \left[\frac{-r^2 e^{-100r}}{100} - 2\left\{\frac{r e^{-100r}}{10^4} + \frac{e^{-100r}}{10^6}\right\}\right]_0^{0.01}$$

$= 15 \cancel{\times 85 \ldots}$ $= \cancel{\frac{2}{\pi 10}}$

$$= 10^{-5} [2\pi] [\pi] \left[3.678 \times 10^{-7} - 2\left\{3.678 \times 10^{-7} + 3.678 \times 10^{-7} - 10^{-6}\right\}\right]$$

$$= 10^{-5} \times 2\pi \times \pi \times 3.839 \times 10^{-6} = 15.85 \times 10^{-12} C$$

$$= 15.85 pC$$

$$\int f(x) g(x) dx = f(x) \int g(x) dx - \int \left(f'(x) \int g(x) dx\right) dx$$

ILATE

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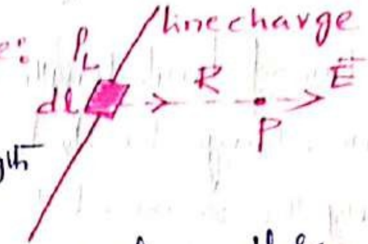
Electric field intensity due to charge distributions.

a) Electric field intensity due to a line charge:



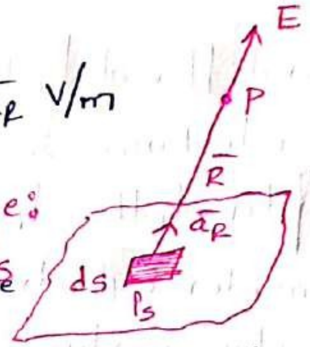
Consider a line charge of charge density ρ_L c/m along a line length

The differential charge dq on a differential dl is $dq = \rho_L dL$

The differential electric field at a point p having unit vector $\bar{a}_R$ from dL is

$$d\vec{E} = \frac{dq}{4\pi\epsilon R^2}\bar{a}_R \triangleq \frac{\rho_L dL}{4\pi\epsilon R^2}\bar{a}_R$$

$$\vec{E} = \int d\bar{E} = \int \frac{\rho_L dL}{4\pi\epsilon R^2}\bar{a}_R = \frac{1}{4\pi\epsilon}\int \frac{\rho_L dL}{R^2}\bar{a}_R \text{ V/m}$$



b) Electric field intensity due to a surface charge:

Consider a surface charge of charge density ρ_s c/m² over a surface 'S'

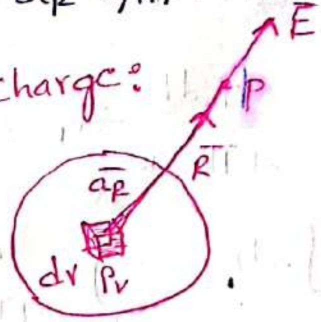
The differential charge dq on the differential ds is $dq = \rho_s ds$

The differential electric field intensity at point p having unit vector $\bar{a}_R$ from ds

$$d\bar{E} = \frac{dq}{4\pi\epsilon R^2}\bar{a}_R = \frac{\rho_s ds}{4\pi\epsilon R^2}\bar{a}_R$$

$$\bar{E} = \int_S d\bar{E} = \int \frac{\rho_s ds}{4\pi\epsilon R^2}\bar{a}_R = \frac{1}{4\pi\epsilon}\int \frac{\rho_s ds}{R^2}\bar{a}_R \text{ V/m}$$

c) Electric field intensity due to a volume charge:



Consider a volume charge of charge density ρ_v c/m3 in a volume V as shown

The differential charge dq in the volume dv is $dq = \rho_v dv$

The differential electric field intensity at point p due to at a distance R is volume charge is $d\bar{E} = \frac{dq}{4\pi\epsilon R^2}\bar{a}_R = \frac{\rho_v dv}{4\pi\epsilon R^2}\bar{a}_R$

$$\bar{E} = \int_V d\bar{E} = \int \frac{\rho_v dv}{4\pi\epsilon R^2}\bar{a}_R = \frac{1}{4\pi\epsilon}\int \frac{\rho_v dv}{R^2}\bar{a}_R \text{ V/m}$$

IES-15 A charge 'Q' is divided between two point charges. what should be the values of this charge on the objects so that the force betwen them is maximum? $\boxed{\text{Ans: } Q/2}$

IES-11 If the potential $v = 4x+2$ volts, electric field is $\boxed{\text{Ans: } -4\bar{a}_x \text{ V/m}}$

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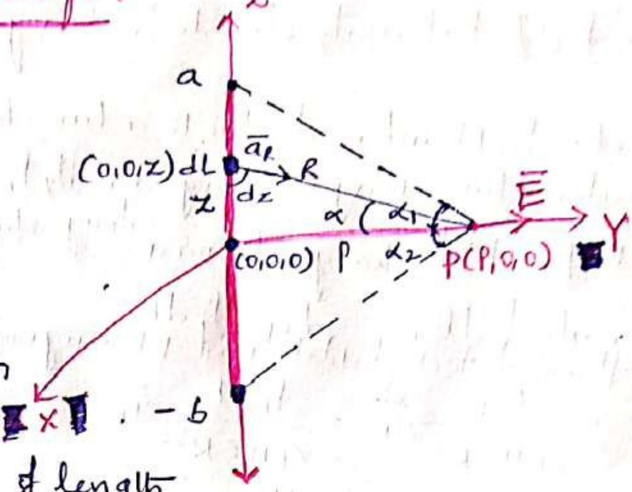
Electric field due to A finite line charge:



Consider a line of finite length l with uniform charge density ρ_L placed along the z-axis. let the end points of the line be at distances a and $-b$ from the origin

Consider a differential charge dQ of length dL at a distance z from the origin, as shown in figure.

Then $\boxed{dQ = \rho_L dL = \rho_L dz}$

Consider cylindrical coordinates. The position of dQ is given by coordinates $(0,0,z)$ and that of point P by $(\rho,0,0)$.

The distance vector between the points $(\rho,0,0)$ and $(0,0,z)$ is

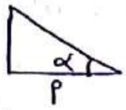
$$\vec{R} = (\rho-0)\bar{a}_\rho + (0-0)\bar{a}_\phi + (0-z)\bar{a}_z = (\rho,0,0)-(0,0,z) = \rho\bar{a}_\rho - z\bar{a}_z$$

The unit vector is $\bar{a}_R = \dfrac{\bar{R}}{|R|} = \dfrac{\rho\bar{a}_\rho - z\bar{a}_z}{\sqrt{\rho^2+z^2}}$

$$d\bar{E} = \frac{\rho_L dL}{4\pi\epsilon R^2}\bar{a}_R = \frac{\rho_L dz\,(\rho\bar{a}_\rho - z\bar{a}_z)}{4\pi\epsilon\left(\sqrt{\rho^2+z^2}\right)^3}$$

$$\bar{E} = \int d\bar{E} = \int_{-b}^{a} \frac{\rho_L dz\,(\rho\bar{a}_\rho - z\bar{a}_z)}{4\pi\epsilon(\rho^2+z^2)^{3/2}} = \frac{\rho_L}{4\pi\epsilon}\int_{-b}^{a}\frac{\rho\bar{a}_\rho\,dz}{(\rho^2+z^2)^{3/2}} - \frac{\rho_L}{4\pi\epsilon}\int_{-b}^{a}\frac{z\bar{a}_z\,dz}{(\rho^2+z^2)^{3/2}}$$

$z = \rho\tan\alpha$, $\quad a = \rho\tan\alpha_1$, $\quad -b = +\rho\tan\alpha_2$, $\quad \alpha_1 = \tan^{-1}(a/\rho)$, $\quad \alpha_2 = \tan^{-1}(-b/\rho)$



$dz = \rho\sec^2\alpha\,d\alpha$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon}\int_{\alpha_2}^{\alpha_1}\frac{\rho^2\sec^2\alpha\,d\alpha\,\bar{a}_\rho}{(\rho^2+\rho^2\tan^2\alpha)^{3/2}} - \frac{\rho_L}{4\pi\epsilon}\int_{\alpha_2}^{\alpha_1}\frac{\rho^2\tan\alpha\,\sec^2\alpha\,d\alpha\,\bar{a}_z}{(\rho^2+\rho^2\tan^2\alpha)^{3/2}}$$

$$= \frac{\rho_L}{4\pi\epsilon}\int_{\alpha_2}^{\alpha_1}\frac{\rho^2\sec^2\alpha\,d\alpha\,\bar{a}_\rho}{\rho^3\sec^3\alpha} - \frac{\rho_L}{4\pi\epsilon}\int_{\alpha_2}^{\alpha_1}\frac{\rho^2\tan\alpha\cdot\sec^2\alpha\,d\alpha\,\bar{a}_z}{\rho^3\sec^3\alpha}$$

$$= \frac{\rho_L}{4\pi\epsilon\rho}\int_{\alpha_2}^{\alpha_1}\cos\alpha\,d\alpha\,\bar{a}_\rho - \frac{\rho_L}{4\pi\epsilon\rho}\int_{\alpha_2}^{\alpha_1}\sin\alpha\,d\alpha\,\bar{a}_z$$

$$\boxed{\bar{E} = \frac{\rho_L}{4\pi\epsilon\rho}\left[(\sin\alpha_1-\sin\alpha_2)\bar{a}_\rho + (\cos\alpha_1-\cos\alpha_2)\bar{a}_z\right]}$$

~~Note~~ Case 1: For an infinite line, $\alpha_1 = \pi/2$, $\alpha_2 = -\pi/2$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(\sin\pi/2 - \sin(-\pi/2))\bar{a}_\rho + (\cos\pi/2 - \cos(-\pi/2))\bar{a}_z\right]$$

$$= \frac{\rho_L}{4\pi\varepsilon\rho}\left[(1+1)\bar{a}_\rho + (0-0)\bar{a}_z\right] = \frac{\rho_L a_\rho}{2\pi\varepsilon\rho} \text{ V/m or N/C}$$

Case 2: If the point P is on the perpendicular bisector of the line charge i.e. a = b, then $\alpha_2 = -\alpha_1$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(\sin\alpha_1 - (\sin(-\alpha_1)))\bar{a}_\rho + (\cos\alpha_1 - \cos(-\alpha_1))\bar{a}_z\right]$$

$$= \frac{\rho_L \sin\alpha_1 a_\rho}{2\pi\varepsilon\rho} \text{ V/m or N/C}$$

Case 3: for a semi-infinity line, that is line extends from

(a) 0 to infinity, and $\alpha_1 = \pi/2, \alpha_2 = 0$ (b) 0 to -infinity

(a) $\alpha_1 = \pi/2$, $\alpha_2 = 0$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(\sin\pi/2 - \sin 0)\bar{a}_\rho + (\cos\pi/2 - \cos 0)\bar{a}_z\right]$$

$$= \frac{\rho_L}{4\pi\varepsilon\rho}\left[\bar{a}_\rho - \bar{a}_z\right] \text{ V/m or N/C}$$

(b) $\alpha_1 = 0$, $\alpha_2 = -\pi/2$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(0 - \sin(-\pi/2))\bar{a}_\rho + (\cos(0) - \cos(-\pi/2))\bar{a}_z\right]$$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[\bar{a}_\rho + \bar{a}_z\right] \text{ V/m or N/C}$$

1) (a) A uniform line charge, infinite in extent, with $\rho_L = 25$ nC/m, is lying on the line x = -3 and z = +4 m in free space. Find the electric field intensity at point (2, 5, 3).

$\rho_L = 25\times10^{-9}$ C/m

Ans: The distance vector between points (2, 5, 3) and (-3, 5, 4) is

$$\bar{R} = 5\bar{a}_x - \bar{a}_z = (2,5,3) - (-3,5,4)$$

Since the line charge is along the y-axis, there is no $\bar{a}_y$ component

$$R = |\bar{R}| = \sqrt{(5)^2 + (-1)^2} = \sqrt{26}$$

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 R}\bar{a}_R = \frac{25\times10^{-9}\times18\times10^{9}}{\sqrt{26}}\frac{[5\bar{a}_x - \bar{a}_z]}{\sqrt{26}}$$

$$= 17.30[5\bar{a}_x - \bar{a}_z] = 86.5\bar{a}_x - 17.3\bar{a}_z \text{ V/m or N/C}$$

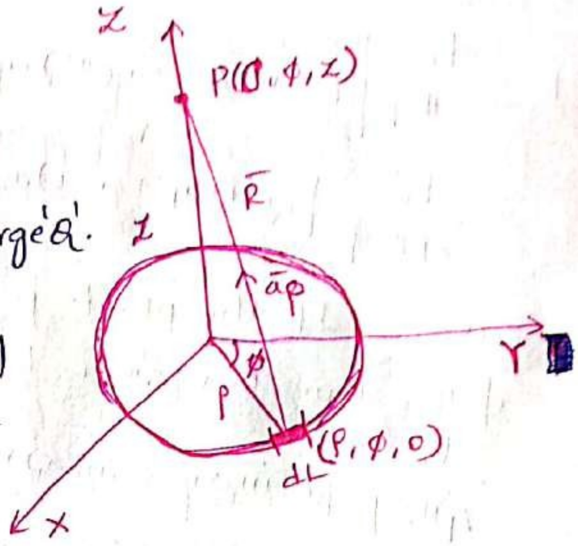
(b) An infinite length of uniform line charge has $\rho_L = 10$ pC/m and it lies along the z-axis. Determine the electric field E at (4, 3, 3) m.

$\rho = \sqrt{x^2+y^2} = \sqrt{4^2+3^2} = 5$ m

Ans:
$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\bar{a}_\rho = \frac{10\times10^{-12}\times18\times10^{9}}{5}a_\rho = 180\times10^{-3}a_\rho$$

$$= 180\, a_\rho \text{ mV/m}$$

Electricfield strength due to a Circular Ring of charge:



Consider a Circular ring of radius ρ placed in x-y plane with its Centre at the origin, carrying charge 'Q'. Let the charge density be ρ_L c/m. Since we are dealing with cylindrical symmetry, the cylindrical Coordinate system is chosen.

Let a point $p(0,\phi,z)$ be on the z-axis, perpendicular to the ring plane as shown in Figure.

Let dq be the differential charge on a differential length dl on the circular path $dq = \rho_L dL$.

The position vector of dq has Coordinates $(\rho,\phi,0)$

$$dl = \rho\, d\phi$$

The distance vector $\vec{R} = (0,\phi,z) - (\rho,\phi,0)$

$$= -\rho\bar{a}_\rho + z\bar{a}_z$$

$$R = |\bar{R}| = \sqrt{\rho^2+z^2} \Rightarrow \bar{a}_R = \frac{-\rho\bar{a}_\rho + z\bar{a}_z}{\sqrt{\rho^2+z^2}}$$

$$d\bar{E} = \frac{\rho_L\, dL}{4\pi\epsilon R^2}\bar{a}_R = \frac{\rho_L\,\rho\, d\phi}{4\pi\epsilon(\rho^2+z^2)}\frac{(-\rho\bar{a}_\rho + z\bar{a}_z)}{(\sqrt{\rho^2+z^2})}$$

$$d\bar{E} = \frac{-\rho_L\rho^2 d\phi\,\bar{a}_\rho}{4\pi\epsilon(\rho^2+z^2)^{3/2}} + \frac{\rho_L\,\rho z\, d\phi}{4\pi\epsilon(\rho^2+z^2)^{3/2}}\bar{a}_z$$

The radial Components of $d\bar{E}$ at point P are symmetrical about x-y plane and are cancelled out. Thus the Component $\bar{a}_\rho$, is zero.

$$d\bar{E} = \frac{\rho_L\,\rho z\,\bar{a}_z}{4\pi\epsilon(\rho^2+z^2)^{3/2}}d\phi \Rightarrow \bar{E} = \int d\bar{E} = \int_{\phi=0}^{2\pi}\frac{\rho_L\,\rho z\, a_z}{4\pi\epsilon(\rho^2+z^2)^{3/2}}d\phi$$

$$= \frac{\rho_L\,\rho z\, a_z}{4\pi\epsilon(\rho^2+z^2)^{3/2}}(2\pi) = \frac{\rho_L\,\rho z}{2\epsilon(\rho^2+z^2)^{3/2}}\bar{a}_z \text{ V/m}$$

If $z=0$, the Electric field strength $\bar{E}=0$. If a point charge is kept at the centre of the ring on the same plane, the force on the point charge is zero.

$$\bar{E} = \frac{\rho_L \rho z}{2\varepsilon(\rho^2+z^2)^{3/2}} \bar{a}_z$$

ρ - radius of the ring is 'a'
$z = h$, distance between point p and centre of the ring.

$$\bar{E} = \frac{\rho_L a h \, \bar{a}_z}{2\varepsilon(h^2+a^2)^{3/2}}$$

what values of h gives the maximum value of $\bar{E}$?

$$\frac{d\bar{E}}{dh} = \frac{\rho_L a}{2\epsilon}\left[\frac{(h^2+a^2)^{3/2}\cdot 1 - h\cdot\frac{3}{\cancel{2}}\cdot(h^2+a^2)^{1/2}\cdot\cancel{2}h}{(h^2+a^2)^{3/2}}\right]$$

$$\boxed{d\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}}$$

$$\frac{d\bar{E}}{dh} = 0$$

$$\frac{\rho_L a}{2\epsilon}\left[\frac{(h^2+a^2)^{3/2} - 3h^2(h^2+a^2)^{1/2}}{(h^2+a^2)^{3/2}}\right] = 0$$

$$(h^2+a^2)^{1/2}\left[h^2+a^2-3h^2\right] = 0$$

$$(h^2+a^2)^{1/2} = 0$$

$$h^2+a^2 = 0 \times$$

$\boxed{a = \pm jh}$ (imaginary values not possible)

$$h^2+a^2-3h^2 = 0$$
$$a^2-2h^2 = 0$$
$$a^2 = 2h^2$$
$$h^2 = a^2/2$$
$$\boxed{h = \frac{\pm a}{\sqrt{2}}}$$

If the total charge on the ring is Q, find E as $a \to 0$

$$\rho_L = \frac{Q}{L} = \frac{Q}{2\pi a}$$

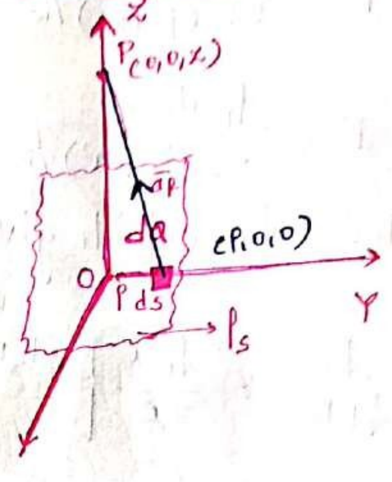
$\boxed{2\pi a = \text{circumference of the ring}}$

$$\bar{E} = \frac{\frac{Q}{2\pi \cancel{a}}\cdot \cancel{a}\cdot h}{2\varepsilon(h^2+a^2)^{3/2}}\bar{a}_z = \frac{Qh}{4\pi\varepsilon(h^2+a^2)^{3/2}}\bar{a}_z$$

As $a \to 0$ $\quad \bar{E} = \frac{Q}{4\pi\varepsilon h^2}\bar{a}_z$ or in general which is the same as that of a point charge as one would expect.

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon R^2}\bar{a}_R}$$

Electric field strength due to an infinite sheet of charge:



Consider an infinite sheet of charge placed in the x-y plane with uniform charge density ρ_s C/m^2. Let the differential charge on a differential surface area ds on the x-y plane be dQ, and let point P be a point at a distance z on the z-axis as shown in figure.

Using cylindrical coordinates, the position vector of dQ has coordinates $(\rho,0,0)$ and that of P is $(0,0,z)$

The differential charge is $\boxed{dQ = \rho_s ds = \rho_s \rho d\rho d\phi}$

The distance vector $\bar{R} = (0,0,z) - (\rho,0,0) = -\rho \bar{a_\rho} + z\bar{a_z}$

$$R = |\bar{R}| = \sqrt{\rho^2+z^2}$$

and the unit vector is $\bar{a_R} = \frac{\bar{R}}{|\bar{R}|} = \frac{-\rho\bar{a_\rho} + z\bar{a_z}}{\sqrt{\rho^2+z^2}}$

We know that the differential field strength is

$$d\bar{E} = \frac{\rho_s ds}{4\pi\varepsilon R^2}\bar{a_R} = \frac{\rho_s \rho d\rho d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}}(-\rho\bar{a_\rho} + z\bar{a_z})$$

$$= \frac{\rho_s \rho d\rho d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}}(-\rho\bar{a_\rho}) + \frac{\rho_s \rho d\rho d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}} z\bar{a_z}$$

Since the sheet is symmetrical about the radial distance the $\bar{a_\rho}$ components will be cancelled.

$$d\bar{E} = \frac{\rho_s \rho d\rho d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}} z\bar{a_z} \Rightarrow \bar{E} = \int_S d\bar{E} = \int_0^{2\pi}\int_0^{\infty} \frac{\rho_s \rho d\rho d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}} z\bar{a_z}$$

$$= \frac{\rho_s z\bar{a_z}}{4\pi\varepsilon}[\phi]_0^{2\pi}\int_0^{\infty}\frac{\rho d\rho}{(\rho^2+z^2)^{3/2}} = \frac{\rho_s z\bar{a_z}}{2\varepsilon}\int_0^{\infty}\frac{\rho d\rho}{(\rho^2+z^2)^{3/2}}$$

$\rho^2+z^2 = t^2$. Then $2\rho d\rho = 2t dt$

$\rho = 0,\ t = z$

$\rho = \infty,\ t = \infty$

$$= \frac{\rho_s z \bar{a_z}}{2\epsilon} \int_z^\infty \frac{t\,dt}{t^3} = \frac{\rho_s z \bar{a_z}}{2\epsilon} \int_z^\infty 1/t^2\,dt$$

$$= \frac{\rho_s z \bar{a_z}}{2\epsilon} \left[-1/t\right]_z^\infty = \frac{\rho_s z \bar{a_z}}{2\epsilon}\left[-0+\frac{1}{z}\right] = \frac{\rho_s}{2\epsilon}\bar{a_z}\ V/m$$

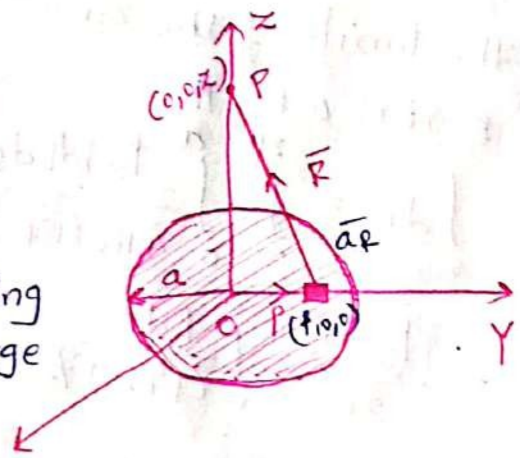
Note1: The field strength has component normal to the plane of sheet charge. In general, if $\bar{a_n}$ is the normal component and if the sheet charge is in any plane, then the electric field strength is $\boxed{\vec{E} = \frac{\rho_s}{2\epsilon}\bar{a_n}\ V/m}$

Note2: If the point p is considered on the negative axis, then the field strength becomes

$$\boxed{\vec{E} = \frac{-\rho_s}{2\epsilon}\vec{a_n}\ V/m}$$

- It is observed that the electric field strength at any point due to an infinite sheet charge is always normal to the plane of the sheet and is independent of the position (distance) of the point.

Electric field strength due to a sheet of Circular disc:



Consider a sheet of circular disc radius 'a' laying in the x-y plane. Let the disc be carrying a charge 'Q' with surface charge density $\rho_s\ C/m^2$. Also, let a differential charge dQ be on a differential surface area 'ds' at a distance ρ on the y-axis, and let point p be at a distance z on the z-axis as shown in figure.

The differential charge is $dQ = \rho_s\,ds$ $\quad ds = \rho\,d\rho\,d\phi$

$$\boxed{dQ = \rho_s\,\rho\,d\rho\,d\phi}$$

The distance vector is $\bar{R} = (0,0,z) - (\rho,0,0)$

$$= -\rho\,\bar{a_\rho} + z\,\bar{a_z}$$

$$R = |\bar{R}| = \sqrt{(-\rho)^2 + (z)^2} = \sqrt{\rho^2 + z^2}$$

unit vector is $\bar{a_R} = \frac{\bar{R}}{|\bar{R}|} = \frac{-\rho\bar{a_\rho} + z\bar{a_z}}{\sqrt{\rho^2+z^2}}$

The differential field strength is

$$d\bar{E} = \frac{\rho_s\,ds}{4\pi\varepsilon R^2}\bar{a_R} = \frac{\rho_s\,\rho\,d\rho\,d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}}(-\rho\bar{a_\rho} + z\bar{a_z})$$

$$= \frac{\rho_s\,\rho\,d\rho\,d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}}(-\rho\bar{a_\rho}) + \frac{\rho_s\,\rho\,d\rho\,d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}}z\bar{a_z}$$

Since the sheet is symmetrical about the radial distance, the $\bar{a_\rho}$ components will be cancelled.

$$d\bar{E} = \frac{\rho_s\,\rho\,d\rho\,d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}}z\bar{a_z}$$

The limits of the circular disc for ρ is 0 to a, and θ it is 0 to 2π

$$\bar{E} = \int_s d\bar{E} = \int_0^{2\pi}\int_0^a \frac{\rho_s\,\rho\,d\rho\,d\phi}{4\pi\varepsilon(\rho^2+z^2)^{3/2}}z\bar{a_z} = \frac{\rho_s z\bar{a_z}}{4\pi\varepsilon}[\phi]_0^{2\pi}\int_0^a \frac{\rho\,d\rho}{(\rho^2+z^2)^{3/2}}$$

$$= \frac{\rho_s z\bar{a_z}}{2\epsilon}\int_0^a \frac{\rho\,d\rho}{(\rho^2+z^2)^{3/2}} \qquad \rho^2+z^2 = t^2$$

$$2\rho\,d\rho = 2t\,dt$$

The limits are at $\rho = 0,\ t = z$

$\rho = a,\ t = \sqrt{a^2+z^2}$

$$\bar{E} = \frac{\rho_s z\bar{a_z}}{2\epsilon}\int_z^{\sqrt{a^2+z^2}} \frac{t\,dt}{t^3} = \frac{\rho_s z\bar{a_z}}{2\epsilon}\left[-\frac{1}{t}\right]_z^{\sqrt{a^2+z^2}} = \frac{\rho_s z\bar{a_z}}{2\epsilon}\left[\frac{-1}{\sqrt{a^2+z^2}} + \frac{1}{z}\right]$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\left[1 - \frac{z}{\sqrt{a^2+z^2}}\right]\bar{a_z}}$$

$z = h$, distance between the point 'p' and centre of the disk

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\left[1 - \frac{h}{(h^2+a^2)^{1/2}}\right]\bar{a_z}\ \ V/m\ (or)\ N/C}$$

$\Rightarrow$ From this derive E field due to an infinite sheet of charge on the z=0 plane.

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\left\{1 - \frac{h}{(h^2+a^2)^{1/2}}\right\}\bar{a_z}$$

For infinite sheet of charge $a \to \infty$

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\{1-0\}\bar{a_z} = \frac{\rho_s}{2\epsilon_0}\bar{a_z}$$

$\Rightarrow$ If $a << h$, show that E is similar to the field due to a point charge.

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\left\{1 - \frac{h}{(h^2+a^2)^{1/2}}\right\}\bar{a_z} = \frac{\rho_s}{2\epsilon_0}\left\{1 - \frac{h}{h(1+a^2/h^2)^{1/2}}\right\}\bar{a_z}$$

$a << h$ so that $a^2/h^2 << 1$

$$= \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{1}{(1+a^2/h^2)^{1/2}}\right]\bar{a_z} = \frac{\rho_s}{2\epsilon_0}\left[1 - \left(1+\frac{a^2}{h^2}\right)^{-1/2}\right]\bar{a_z}$$

$$(1 \pm x)^{-1/2} = 1 \mp \frac{1}{2}x + \cdots$$

$$= \frac{\rho_s}{2\epsilon_0}\left[1 - \left(1 - \frac{1}{2}\frac{a^2}{h^2}\right)\right]\bar{a_z} = \frac{\rho_s}{2\epsilon_0}\cdot\frac{1}{2}\cdot\frac{a^2}{h^2}\bar{a_z} \qquad \boxed{\rho_s = \frac{Q}{\pi a^2}}$$

$$= \frac{\frac{Q}{\pi a^2}\cdot\frac{1}{2}\cdot\frac{a^2}{h^2}}{2\epsilon_0}\bar{a_z} = \frac{Q}{4\pi\epsilon_0 h^2}\bar{a_z} \quad V/m \text{ or } N/C$$

$$\boxed{\bar{E} = \frac{Q}{4\pi\epsilon h^2}\bar{a_z} \quad V/m}$$

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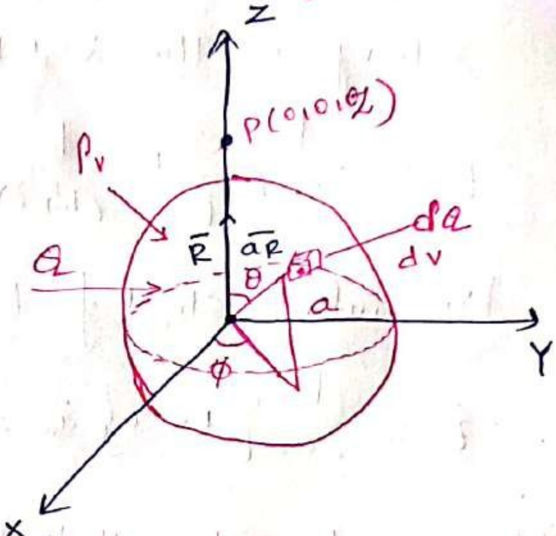
Electric field intensity due to volume charge distribution



Volume charge distribution with uniform charge density ρ_v.

The charge dQ associated with the elemental volume dv is

$$\boxed{dQ = \rho_v dv}$$

and hence the total charge on thin a sphere of radius 'a' is

$$d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\bar{a}_R$$

$$\bar{E} = \int_v d\bar{E} = \int \frac{dQ}{4\pi\varepsilon R^2}\bar{a}_R = \int \frac{\rho_v dv}{4\pi\varepsilon R^2}\bar{a}_R$$

$$\bar{R} = (0,0,z) - (0,0,0) = z\bar{a}_z$$

$$R = |\bar{R}| = z$$

$$= \int_v \frac{\rho_v dv}{4\pi\varepsilon z^2}\bar{a}_R = \frac{a_z}{4\pi\varepsilon z^2}\int_v \rho_v dv$$

In spherical coordinate system

$$\boxed{dv = r^2 \sin\theta\, dr\, d\theta\, d\phi}$$

$$= \int_{r=0}^{a}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 \sin\theta\, dr\, d\theta\, d\phi = \left[\frac{r^3}{3}\right]_0^a \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$= \frac{a^3}{3}\cdot[1+1][2\pi] = \frac{4\pi}{3}a^3 = \text{Volume of the sphere}$$

$$\boxed{\rho_v \cdot v = Q, \text{ charge}}$$

$$\bar{E} = \frac{a_z}{4\pi\varepsilon z^2}[\rho_v \cdot v]$$

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon z^2} a_z \ \text{V/m or N/C}}$$

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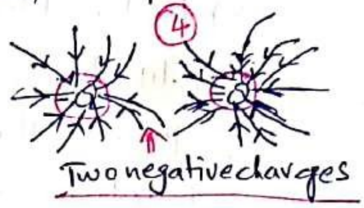
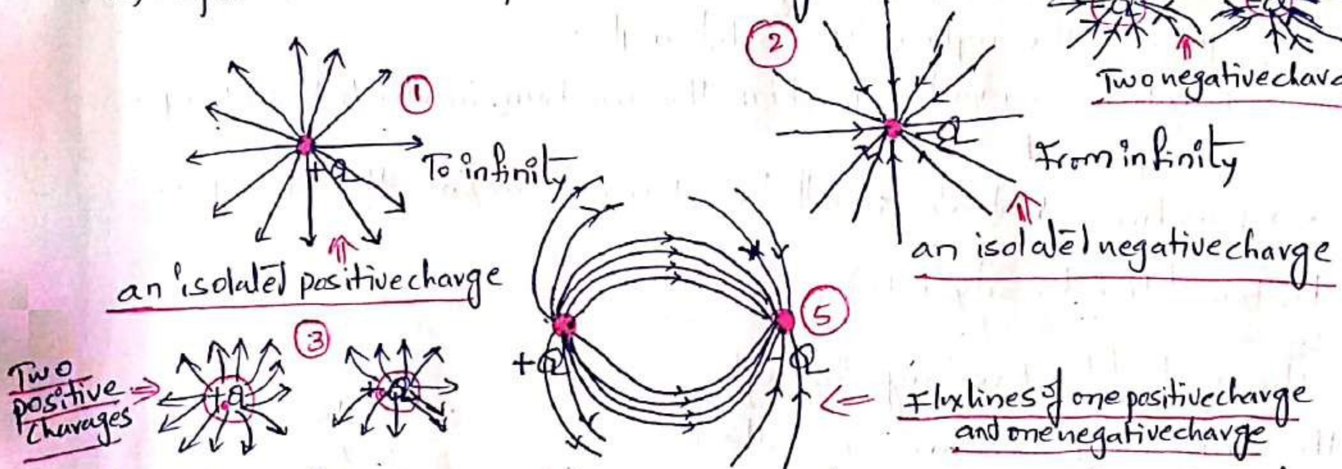
DIFFERENCES BETWEEN CIRCUIT THEORY AND ELECTROMAGNETIC FIELD THEORY

Circuit theory	*Field theory*
1. Deals with voltage (V) and current (I)	Deals with Electric (**E**) and Magnetic (**H**) fields
2. V and I are scalars	**E** and **H** are vectors
3. V and I are produced from **E** and **H** respectively	**E** and **H** are produced from V and I respectively
4. V and I are functions of time (t)	**E** and **H** are functions time (t) and space variables (x, y, z) or (ρ, ϕ, z) or (r, θ, ϕ)
5. Radiation effects are neglected	Radiation effects can be considered
6. Using circuit theory, transmitter and receiver circuits can be analysed and designed. But it cannot be used to design or analyse a medium like free space	Using field theory, the medium also can be designed and analysed
7. This is simplified approximation of field theory	This is a more accurate theory
8. The variables of circuit theory, V and I are integrated effects of variables of field theory **E** and **H**	The variables of field theory, **E** and **H** are integrated effects of variables of circuit theory V and I
9. Circuit theory cannot be used to analyse or design a complete communication system	Field theory can be used where circuit theory fails to hold good for the analysis and design of a communication system
10. Is useful at low frequencies	Is useful at all frequencies, particularly at high frequencies
11. At low frequencies the length of connecting wires is very much smaller than λ	At high frequencies the length of connecting components are of the order of λ
12. Cannot be applied in free space	Is applicable in free space
13. Is simple	Is complex but it is simplified by using appropriate mathematics
14. Basic laws are Ohms law, Kirchoff's laws	Basic laws are Coulomb's law, Gauss's law, Ampere's circuit law
15. Basic theorems are Thevenin's, Nortan's, Reciprocity, Superposition, Maximum power transfer theorems	Basic theorems are Reciprocity, Helmholtz, Stoke's, Divergence and Poynting theorems
16. Basic equations are Mesh/Loop equations	Basic equations are Maxwell, Poission, Laplace and Wave

Electric flux (Ψ) and Electric flux density ($\bar{D}$):

The electric displacement from positive charge to the negative through the medium is called Electric flux. It is the total number of lines of force in the electric field and is represented by the symbol 'Ψ'. The lines of force are called flux lines or stream lines. The amount of displacement of flux is equal in magnitude to the positive charge.





An Electric charge produces electric flux and as the charge increases, the dielectric flux also increases. Therefore, electric flux is numerically equal to the electric charge producing it. If an electric charge produces Q Coulombs, then electric flux, associated with it is given by

$$\boxed{\Psi = Q}$$

- For an isolated positive charge Q, the electric flux originates radially from positive charge and terminates at infinity (1)

Similarly, for an isolated negative charge (-Q), the electric flux from infinity radially terminates at the negative charge as shown in figure (2)

If two like charges are placed near each other, a force of repulsion comes into play. The flux lines from two positive charges repel each other (3) and terminates at infinity. Likewise, the flux lines from infinity repels each other and terminates at negative charges (4)

Consider two point charges placed in space. If one charge (testcharge) is brought near another charge, a force exists between them. That is, if a positive charge is placed near a negative charge, an electric field is developed between the charges due to the force. The lines force originates radially from the positive charge and terminate at the negative charge (5)

→ Electric flux is a scalar quantity

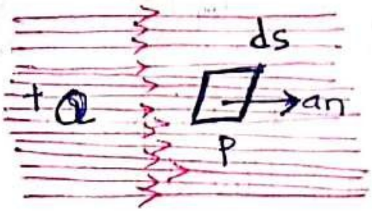
→ Flux lines are always parallel to each other and are equally spaced throughout the solid angle.

→ Flux lines do not depend on the medium in which the charges are placed.

→ Flux lines radiate in all directions from / into the charges.

Electric flux density ($\bar{D}$):

The net flux passing through the unit surface area is called electric flux density $\bar{D}$. The electric flux density is normal to the surface area. It is expressed as



$$\boxed{\bar{D} = \frac{d\psi}{ds}\,\bar{a}_n \quad C/m^2}$$

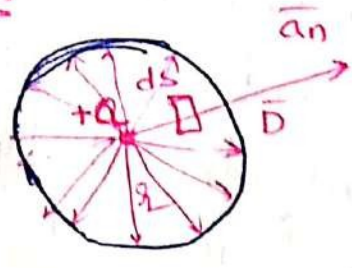
$$\boxed{\psi = \vec{D}\cdot\vec{S}}$$

$$\boxed{\psi = \int D\cdot ds}$$

Electric flux density is also called electric displacement

Electric flux density due to point charge Q:

Consider an imaginary sphere of radius r and a point charge +Q placed at its centre as shown in Figure. The flux lines originating from the point charge distribute radially over the surface of the sphere.



The flux density at a differential surface ds is

$$\bar{D} = \frac{d\psi}{ds}\,\bar{a}_n$$

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If the total surface area through which the flux ψ passes is considered, due to symmetry in the radial direction, $\bar{D}$ is always normal to the surface area.

$$\bar{D} = \frac{\text{Total flux}}{\text{Surface Area of the sphere}} = \frac{\psi}{4\pi r^2}\bar{a_n}$$

$d\bar{s}_r = r\,d\theta\, r\sin\theta\, d\phi$

$= r^2 \sin\theta\, d\theta\, d\phi$

$$S = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 \sin\theta\, d\theta\, d\phi$$

$\Rightarrow r^2 [-\cos\theta]_0^{\pi} [\phi]_0^{2\pi}$

$\Rightarrow r^2 [1+1][2\pi - 0] = 4\pi r^2$

Since $\psi = Q$

$$\bar{D} = \frac{Q}{4\pi r^2}\bar{a_n} \text{ C/m}^2$$

Since $\bar{a_n} = \bar{a_r}$ the radial direction normal to the surface area.

$$\bar{D} = \frac{Q}{4\pi r^2}\bar{a_r}.$$

From above equation

Electric flux density depends on the charge and radial distance, and is independent of the charactestics of the medium.

Relation between $\vec{E}$ and $\vec{D}$:

The electric field strength $\bar{E}$ at a distance r from a positive point charge Q is given by

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\bar{a_r} = \frac{1}{\varepsilon}\cdot\frac{Q}{4\pi r^2}\bar{a_r} = \frac{\bar{D}}{\varepsilon}$$

$$\boxed{\bar{D} = \varepsilon \bar{E}}$$

E is a function of the permittivity of the medium, which the flux density is not.

→ For a medium where permittivity is constant, the direction of displacement is along the direction of $\bar{E}$.

problem: A point charge $Q = 60\,nC$, is located at the origin of a cartesian coordinate system. Find the electric flux density D at (4, 7, −8).

Solution:

$$\bar{D} = \frac{Q}{4\pi R^2}\bar{a_R}$$

Q (0,0,0) → (4,7,−8) D

$\bar{R} = (4,7,-8) - (0,0,0) = 4\bar{a_x} + 7\bar{a_y} - 8\bar{a_z}$

$$\bar{a_R} = \frac{4\bar{a_x} + 7\bar{a_y} - 8\bar{a_z}}{\sqrt{4^2+7^2+(-8)^2}} = \frac{4\bar{a_x} + 7\bar{a_y} - 8\bar{a_z}}{\sqrt{129}}$$

$R = |\bar{R}| = \sqrt{129}$

$$= \frac{60\times 10^{-9}}{4\pi(\sqrt{129})^2}\cdot\frac{(4\bar{a_x} + 7\bar{a_y} - 8\bar{a_z})}{\sqrt{129}}$$

$= (13\bar{a_x} + 22.8\bar{a_y} - 26\bar{a_z})$ pC/m^2

Electric flux density due to charge distributions:

case 1: For an infinite line of charge density ρ_L. The electric field is given by the equation

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\bar{a_\rho}$$

$$\boxed{\bar{D} = \varepsilon\bar{E} = \frac{\rho_L}{2\pi\rho}\bar{a_\rho} \ c/m^2}$$

case 2: For an infinite sheet of charge density ρ_s c/m^2. The electric field is given by the equation

$$\bar{E} = \frac{\rho_s}{2\epsilon}\bar{a_n}$$

an is the unit vector normal to the plane of the sheet.

$$\boxed{\bar{D} = \varepsilon\bar{E} = \frac{\rho_s}{2}\bar{a_n} \ c/m^2}$$

Case 3: For a volume charge enclosed by a sphere of radius r, having a uniform charge density ρ_v c/m^3

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\bar{a_r}$$

$$\bar{D} = \varepsilon\bar{E} = \frac{Q}{4\pi r^2}\bar{a_r} \quad \text{where } Q = \int \rho_v dv$$

$$\boxed{\bar{D} = \int_v \frac{\rho_v dv}{4\pi r^2}\bar{a_r} \ c/m^2}$$

The Divergence theorem states that the total outward flux of a vector field A through the closed surface S is the same as the volume integral of the divergence of A.

$$\oint_S A\cdot ds = \int_V (\nabla\cdot\bar{A})dV$$

Divergence theorem also known as Gauss-Ostrogradsky theorem

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Gauss's law [Maxwell's First law]:

Karl Friedrich Gauss (1777-1855), a German mathematician, developed the divergence theorem. He was the first physicist to measure electric and magnetic quantities in absolute units.

Gauss's law states that the "total electric flux ψ through any closed surface is equal to the total charge enclosed by that surface." $\psi = Q_{enclosed}$

$\boxed{\text{closed surface also known as Gaussian Surface}}$

$$\psi = \oint d\psi = \oint D \cdot ds = Q_{enclosed} = \int \rho_v \, dv$$

$$\boxed{Q = \oint D \cdot ds = \int_v \rho_v dv} \Longrightarrow \text{Integral Form}$$

$\longrightarrow$ By applying divergence theorem

$$\oint D \cdot ds = \int \nabla \cdot \bar{D} \, dv$$

$$Q = \int \nabla \cdot \bar{D} \, dv = \int \rho_v \, dv$$

$$\boxed{\nabla \cdot \bar{D} = \rho_v} \Longrightarrow \text{Differential form or point form}$$

The divergence of the flux density is the net flux crossing the surface per unit volume. The divergence theorem relates a volume integration to a surface integration when the volume of charge enclosed by the closed surface.

Applications of Gauss's law:- 1) It can be applied to many practical problems where the electric field of charge distribution has symmetrical geometry, such as sphere, cylinder or plane.

2) We can easily find the net flux density, electric field and the charge enclosed by the surfaces for such symmetrical charge distributions.

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The divergence of electric flux density in various coordinate systems

a) Cartesian Coordinate system (Rectangular coordinate system):

$$\boxed{\nabla \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}}$$

$\bar{D} = \bar{D_x}\,\bar{a_x} + D_y\,\bar{a_y} + D_z\,\bar{a_z}$

b) Cylindrical coordinate system:

$$\boxed{\nabla \cdot \bar{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}}$$

c) Spherical coordinate system:

$$\boxed{\nabla \cdot \bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}}$$

~~Idems:~~ ~~The finite street~~ limitations of Gauss law:

1. It cannot be applied on non-Gaussian surfaces
2. It can be applied only if the surface encloses the volume completely.
3. It does not specify any particular shape for the closed surface
4. It cannot be applied if the charges are outside the surface

problem:
The finite sheet $0 \le x \le 1$, $0 \le y \le 1$ on the $z=0$ plane has a charge density $\rho_s = xy(x^2+y^2+25)^{3/2}$ nc/m². Find
(a) Total charge on the sheet
(b) The electricfield at $(0,0,5)$
(c) The force experienced by a -1mc charge located at $(0,0,5)$.

solution: (a) $$Q = \int \rho_s\, ds = \int_{x=0}^{1}\int_{y=0}^{1} xy(x^2+y^2+25)^{3/2}\,dx\,dy \times 10^{-9}$$

$$= 10^{-9}\int_{y=0}^{1} y\,dy \int_{x=0}^{1} (x^2+y^2+25)^{3/2}\frac{(2x)}{2}dx = \frac{1}{2}\times 10^{-9}\int_{y=0}^{1} y\,dy\left[\frac{(x^2+y^2+25)^{3/2+1}}{(3/2+1)}\right]_{x=0}^{1}$$

$$\Rightarrow \frac{10^{-9}}{2}\int_{y=0}^{1}\frac{2y}{5}dy\left[(x^2+y^2+25)^{5/2}\right]_{x=0}^{1} = \frac{10^{-9}}{2}\int_{y=0}^{1}\frac{2y}{5}dy\left[(y^2+26)^{5/2} - (y^2+25)^{5/2}\right]$$

$$\Rightarrow \frac{10^{-9}}{10}\int_{y=0}^{1}\left[(y^2+26)^{5/2}\cdot(2y)dy - (y^2+25)^{5/2}\cdot(2y)dy\right]$$

$$\Rightarrow \frac{10^{-9}}{10}\left[\frac{(y^2+26)^{5/2+1}}{(5/2+1)} - \frac{(y^2+25)^{5/2+1}}{(5/2+1)}\right]_{y=0}^{1}$$

$$= \frac{10^{-9}}{10}\left[\frac{2}{7}(27)^{7/2} - \frac{2}{7}(26)^{7/2} - \frac{2}{7}(26)^{7/2} + \frac{2}{7}(25)^{7/2}\right]$$

$$= \frac{10^{-9}}{35}\left[(27)^{7/2} - 2(26)^{7/2} + (25)^{7/2}\right] = \frac{10^{-9}}{35}\times 1160.13 = 33.15\times 10^{-9}$$

$= 33.15$ nC

(b)
$$E = \int \frac{\rho_s\, ds}{4\pi\varepsilon R^2}\bar{a}_R = \int \frac{\rho_s\, ds\, \bar{R}}{4\pi\varepsilon R^3} \qquad ds = dx\,dy$$

Sheet on the $z=0$plane has coordinates $(x,y,0)$. Electricfield at $(0,0,5)$

$$\bar{R} = (0,0,5) - (x,y,0) = (-x,-y,5) = -x\bar{a}_x - y\bar{a}_y + 5\bar{a}_z$$

$$R = |\bar{R}| = \sqrt{x^2+y^2+25}$$

$$= \int_{x=0}^{1}\int_{y=0}^{1}\frac{xy\cancel{(x^2+y^2+25)^{3/2}}\,dx\,dy\cdot(-x\bar{a}_x - y\bar{a}_y + 5\bar{a}_z)\times 10^{-9}}{4\pi\times\frac{10^{-9}}{36\pi}\times\cancel{(x^2+y^2+25)^{3/2}}}$$

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$$= 9\int_{x=0}^{1}\int_{y=0}^{1} xy(-x a_x - y a_y + 5 a_z)\,dx\,dy$$

$$= 9\left[\int_{x=0}^{1}\int_{y=0}^{1} -x^2 y\, a_x\,dx\,dy - \int_{x=0}^{1}\int_{y=0}^{1} x y^2\, a_y\,dx\,dy + \int_{x=0}^{1}\int_{y=0}^{1} 5 a_z\,dx\,dy\right]$$

$$= 9\left[\left(-x^3/3\right)_0^1 \left(y^2/2\right)_0^1 \bar{a_x} - \left(x^2/2\right)_0^1 \left(y^3/3\right)_0^1 \bar{a_y} + 5\left(x^2/2\right)_0^1 \left(y^2/2\right)_0^1 \bar{a_z}\right]$$

$$= 9\left[-\tfrac{1}{6}\bar{a_x} - \tfrac{1}{6}\bar{a_y} + \tfrac{5}{4}\bar{a_z}\right] = -1.5\bar{a_x} - 1.5\bar{a_y} + 11.25\bar{a_z}\ \text{V/m}$$

(c) The force experienced by a -1mc charge located at (0,0,5)

$$\bar{F} = \bar{E}Q$$
$$= -(1.5\bar{a_x} - 1.5\bar{a_y} + 11.25\bar{a_z}) \times (-1\times 10^{-3})$$
$$= (1.5\bar{a_x} + 1.5\bar{a_y} - 11.25\bar{a_z})\times 10^{-3} = (1.5\bar{a_x} + 1.5\bar{a_y} - 11.25\bar{a_z})\,\text{mN}$$

Problem: A square plate described by $-2 \le x \le 2, -2 \le y \le 2, z = 0$ carries a charge $12|Y|$ mc/m². Find the total charge on the plate and the electric field intensity at (0,0,10).

Solution: Total charge on the plate

$$Q = \int \rho_s\,ds = \int_{x=-2}^{2}\int_{y=-2}^{2} 12|Y|\,dx\,dy \times 10^{-3} = 12\int_{x=-2}^{2} dx \int_{0}^{2} 2y\,dy \times 10^{-3}$$

$$= 12\,[x]_{-2}^{2}\,[2\cdot y^2/2]_0^2 \times 10^{-3} = 12[4][4-0]\times 10^{-3} = 192\times 10^{-3} = 192\,\text{mC}$$

Electric field intensity at (0,0,10)

$$\bar{E} = \int \frac{\rho_s\,ds}{4\pi\varepsilon R^2}\bar{a_R} = \int \frac{\rho_s\,ds\,\bar{R}}{4\pi\varepsilon R^3}$$

Square plate on the $z=0$ plane has coordinates $(x,y,0)$.

$$\bar{R} = (0,0,10) - (x,y,0) = (-x,-y,10) = (-x\bar{a_x} - y\bar{a_y} + 10\bar{a_z})$$
$$R = |\bar{R}| = \sqrt{(-x)^2 + (-y)^2 + (10)^2} = \sqrt{x^2 + y^2 + 100}$$

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9$$ 27

$$= \int_{x=-2}^{2}\int_{y=-2}^{2} \frac{12|y|\times10^{-3}\times(-x\bar{a}_x - y\bar{a}_y + 10\bar{a}_z)\times9\times10^{9}}{(x^2+y^2+100)^{3/2}}\,dx\,dy$$

$$= 108\times10^{6}\left[\underbrace{\int_{y=-2}^{2}|y|\int_{x=-2}^{2}\frac{-x\,\bar{a}_x\,dx\,dy}{(x^2+y^2+100)^{3/2}}}_{0} + \underbrace{\int_{x=-2}^{2}x\int_{y=-2}^{2}\frac{y|y|\,\bar{a}_y\,dx\,dy}{(x^2+y^2+100)^{3/2}}}_{0}\right.$$

$$\left. + 10\int_{x=-2}^{2}\int_{y=-2}^{2}\frac{|y|\,dx\,dy\,\bar{a}_z}{(x^2+y^2+100)^{3/2}}\right]$$

$$= 108\times10^{+7}\,\bar{a}_z\int_{x=-2}^{2}\int_{y=-2}^{2}\frac{|y|\,dx\,dy}{(x^2+y^2+100)^{3/2}}$$

$$= 108\times10^{7}\,\bar{a}_z\int_{x=-2}^{2}\int_{y=0}^{2}\frac{2y\,dx\,dy}{(x^2+y^2+100)^{3/2}} \quad = \cancel{216\times10^{7}\times\bar{a}_z\int_{x=-2}^{2}\left[\frac{-1}{(x^2+y^2+100)^{1/2}}\right]_{y=0}^{2}dx}$$

$$= \cancel{108\times10^{7}\times\bar{a}_z} = 216\times10^{7}\,\bar{a}_z\int_{x=-2}^{2}\left[\frac{-1}{(x^2+y^2+100)^{1/2}}\right]_{y=0}^{2}dx$$

$$= -216\times10^{7}\times\bar{a}_z\int_{x=-2}^{2}\left[\frac{1}{(x^2+104)^{1/2}} - \frac{1}{(x^2+100)^{1/2}}\right]dx$$

$$= -216\times10^{7}\,\bar{a}_z\left[\ln\left(x+\sqrt{x^2+104}\right) - \ln\left(x+\sqrt{x^2+100}\right)\right]_{x=-2}^{2}$$

$$= -216\times10^{7}\,a_z\left[\ln\left(\frac{x+\sqrt{x^2+104}}{x+\sqrt{x^2+100}}\right)\right]_{x=-2}^{2}$$

$$= -216\times10^{7}\,a_z\left[\ln\left(\frac{2+\sqrt{108}}{2+\sqrt{104}}\right) - \ln\left(\frac{-2+\sqrt{108}}{-2+\sqrt{104}}\right)\right]$$

$$= -216\times10^{7}\,a_z\left[0.015800 - 0.02342\right]$$

$$= -216\times10^{7}\,\bar{a}_z\times(-7.62\times10^{-3}) = 16.4592\times10^{6}\,a_z$$

$$= 16.452\,\bar{a}_z\ \text{MV/m}$$

problem: planes $x=2, y=-3$ respectively, carry charges $10 nc/m^2$ and $15 nc/m^2$. If the line $x=0, z=2$ carries charge $10\pi nc/m$, Calculate E at (1,1,-1) due to the three charge distributions.

Solution:
1. $x=2$ Carries charge $\rho_s = 10 nc/m^2$ (surface charge density)
2. $y=-3$ Carries charge $\rho_s = 15 nc/m^2$ (surface charge density)
3. $x=0, z=2$ Carries charge $\rho_L = 10\pi nc/m$ (line charge density)

$$\boxed{\frac{1}{2\epsilon_0} = 18\pi \times 10^9}$$

at $\rightarrow \bar{E}_1 = \frac{\rho_s}{2\epsilon_0}(-\bar{a_x})$

$= 18\pi \times 10^9 \times 10 \times 10^{-9} (-\bar{a_x}) = -180\pi a_x$ V/m

$\rightarrow \bar{E_2} = \frac{\rho_s}{2\epsilon_0}(\bar{a_y}) = 18\pi \times 10^9 \times 15 \times 10^{-9} (\bar{a_y}) = 270\pi \bar{a_y}$ V/m

$$\boxed{\frac{1}{2\pi\epsilon_0} = 18 \times 10^9}$$

$\rightarrow \bar{E_3} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\bar{a_\rho} = \frac{\rho_L}{2\pi\epsilon_0 R}\bar{a_R}$

line charge at $x=0, z=2$ has no y-component

$\bar{R} = (1,1,-1) - (0,1,2)$
(or)
$= (1,0,-1) - (0,0,2) = (1,0,-3) = (\bar{a_x} - 3\bar{a_z})$

$R = |\bar{R}| = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$

$\bar{E_3} = \frac{\cancel{10}\pi \times \cancel{10^{-9}} \times 18 \times \cancel{10^9}}{\sqrt{10}} \times \frac{(\bar{a_x} - 3\bar{a_z})}{\sqrt{10}} = 18\pi(\bar{a_x} - 3\bar{a_z})$ V/m

Now the total electric field intensity due to three distributions

$\bar{E} = \bar{E_1} + \bar{E_2} + \bar{E_3} = -180\pi\bar{a_x} + 270\pi\bar{a_y} + 18\pi\bar{a_x} - 54\pi\bar{a_z}$

$$\boxed{\bar{E} = -162\pi\bar{a_x} + 270\bar{a_y} - 54\pi\bar{a_z} \text{ V/m}}$$

problem: Determine D at (4,0,3) if there is a point charge $-5\pi mc$ at (4,0,0) and a line charge $3\pi mc/m$ along the Y-axis.

solution: Let D is the sum of flux densities due to a point charge (D_Q) and line charge (D_L)

$D_Q = \epsilon_0 \bar{E} = \epsilon_0 \times \frac{Q}{4\pi\epsilon_0 R^2}\bar{a_R} = \frac{Q}{4\pi R^2}\bar{a_\rho} = \frac{Q \cdot \bar{R}}{4\pi R^3}$

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$\bar{R} = (4,0,3) - (4,0,0) = (0,0,3) = 3\bar{a_z}$

$R = |\bar{R}| = \sqrt{3^2} = 3$

$\rightarrow D_q = \dfrac{-5\pi \times 10^{-3} \times 3\bar{a_z}}{(3)^3 \times 4\pi} = -0.138\,\bar{a_z}\ mc/m^2$

$\rightarrow D_L = \dfrac{\rho_L}{2\pi\rho}\bar{a_\rho} = \dfrac{\rho_L}{2\pi R}\bar{a_R}$ ρ_L along y-axis

$\bar{R} = (4,0,3) - (0,0,0) = (4,0,3) = 4\bar{a_x} + 3\bar{a_z}$

$R = |\bar{R}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

$D_L = \dfrac{3\pi \times 10^{-3}}{2\pi(5)}\dfrac{(4a_x + 3a_z)}{5} = 0.24\bar{a_x} + 0.18\bar{a_z}\ mc/m^2$

Now the total electric flux density $D = D_q + D_L$

$= -0.138\,\bar{a_z} \times 10^{-3} + 0.24\bar{a_x} \times 10^{-3} + 0.18\bar{a_z} \times 10^{-3}$

$\boxed{D = 0.24\bar{a_x} - 0.042\,\bar{a_z}\ mc/m^2}$

Problem: A point charge of 30nC is located at the origin while plane y=3 carries charge 10nC/m². Find D at (0,4,3)

Solution Let D is the sum of electric flux density due to point charge and charge density

$D = D_q + D_s$

$\bar{R} = (0,4,3) - (0,0,0) = (0,4,3)$

$\bar{R} = 4\bar{a_y} + 3\bar{a_z}$

$R = |\bar{R}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

$\rightarrow D_q = \dfrac{Q}{4\pi R^2}\bar{a_R} = \dfrac{Q\,\bar{R}}{4\pi R^3}$

$= \dfrac{30 \times 10^{-9} \times (4\bar{a_y} + 3\bar{a_z})}{4\pi(5)^3} = \dfrac{30}{500\pi}(4\bar{a_y} + 3\bar{a_z})\ nc/m^2$

$\rightarrow D_s = \dfrac{\rho_s}{2}\bar{a_n} = \dfrac{\rho_s}{2}\bar{a_y} = \dfrac{10 \times 10^{-9}}{2}\bar{a_y} = 5\bar{a_y}\ nc/m^2$

Now the total electric flux density $D = D_q + D_s$

$= \left(\dfrac{30}{500\pi}(4\bar{a_y}) + \dfrac{30}{500\pi}(3\bar{a_z})\right) \times 10^{-9} + 5\bar{a_y} \times 10^{-9}$

$\boxed{D = 5.076\bar{a_y} + 0.057\,\bar{a_z}\ nc/m^2}$

Problem

Given that $D = z\rho \cos^2\phi \,\bar{a}_z$ C/m², calculate the charge density at $(1, \pi/4, 3)$ and the total charge enclosed by the cylinder of radius 1m with $-2 \le z \le 2$ m.

Solution:

$$\nabla\cdot\bar{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\bar{D} = \rho_v = \frac{\partial}{\partial z}(z\rho\cos^2\phi) = \rho\cos^2\phi$$

$$\nabla\cdot\bar{D} = \rho_{v\,at}(1, \pi/4, 3) \qquad \rho = 1,\ \phi = \pi/4,\ z = 3$$

$$= 1\cdot\cos^2(\pi/4) = \tfrac{1}{2} = 0.5\ \text{C/m}^3$$

Cylindrical coordinate system
$dv = \rho\, d\rho\, d\phi\, dz$

$$Q = \int_v \rho_v\, dv$$

$$= \int_{z=-2}^{2}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{1} \rho\cos^2\phi\, \rho\, d\rho\, d\phi\, dz = \int_{z=-2}^{2} dz \int_{\phi=0}^{2\pi} \cos^2\phi\, d\phi \int_{\rho=0}^{1} \rho^2 d\rho$$

$$= [z]_{-2}^{2}\cdot\frac{1}{2}\left[\phi + \frac{\sin 2\phi}{2}\right]_0^{2\pi}\left[\rho^3/3\right]_0^1$$

$$= 4\cdot\frac{1}{2}[2\pi]\left[\frac{1}{3}\right] = \frac{4\pi}{3}\ \text{C}$$

$$\int_{\phi=0}^{2\pi}\left(\frac{1+\cos 2\phi}{2}\right)d\phi = \frac{1}{2}\left[\phi + \frac{\sin 2\phi}{2}\right]_0^{2\pi} = \frac{1}{2}[(2\pi - 0) + (0-0)] = \pi$$

Problem:

If $D = (2y^2 + z)\bar{a}_x + 4xy\,\bar{a}_y + x\,\bar{a}_z$ C/m²

find (a) volume charge density at $(-1, 0, 3)$

(b) The flux through the cube defined by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$

(c) The total charge enclosed by the cube.

Solution

(a) $$\nabla\cdot\bar{D} = \rho_v \Rightarrow \left(\frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z\right)\left[(2y^2+z)\bar{a}_x + 4xy\,\bar{a}_y + x\,\bar{a}_z\right]$$

$$\rho_v = 0 + 4x + 0 = 4x$$

$$\rho_v \text{ at } (-1, 0, 3) = -4\ \text{C/m}^3$$

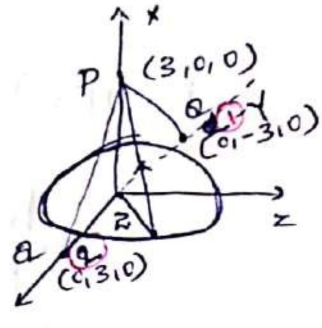
Cartesian Coordinate system
$dv = dx\,dy\,dz$

(b) $$\Psi = Q = \int \rho_v\, dv = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} +4x\, dx\,dy\,dz = +4(x^2/2)_0^1 (y)_0^1 (z)_0^1 = 2\ \text{C}$$

(c) The total charge enclosed by the cube is equal to the net flux as per Gauss law $Q = 2$ C

Problem: A ring placed along $y^2+z^2=4$, $x=0$ carries a uniform charge of $5\,\mu C/m$ (a) Find D at P(3,0,0)

(b) If two identical point charges are placed at (0,-3,0) and (0,3,0) in addition to the ring, find the value of Q such that D = 0 at P.



Solution: (a)

$$\bar{E} = \frac{\rho_L a h}{2\epsilon(h^2+a^2)^{3/2}}\bar{a_x}$$

$y^2+z^2=4 \Rightarrow y^2+z^2=2^2$

$(y-0)^2+(z-0)^2=2^2$

a = radius = 2 m

h = distance between the center of the ring and point p = 3m

$\rho_L = 5\mu C/m$

$$\bar{E} = \frac{5\times10^{-6}\times2\times3}{2\epsilon(2^2+3^2)^{3/2}}\bar{a_x} \Rightarrow \epsilon\bar{E} = \bar{D_L} = \frac{5\times10^{-6}\times2\times3}{2(4+9)^{3/2}}\bar{a_x}$$

$\bar{D} = 0.64\,\bar{a_x}\,\mu C/m^2$

(b) Solution:

$$\bar{D} = \frac{Q}{4\pi R^2}\bar{a_R} = \frac{Q\bar{R}}{4\pi R^3}$$

Consider the point charge is Q

$$\bar{D_p} = \bar{D_1}+\bar{D_2}$$

$$\Rightarrow \frac{Q[(3,0,0)-(0,-3,0)]}{4\pi|(3,0,0)-(0,-3,0)|^3} + \frac{Q[(3,0,0)-(0,3,0)]}{4\pi|(3,0,0)-(0,3,0)|^3}$$

$$\Rightarrow \frac{Q(3,3,0)}{4\pi(18)^{3/2}} + \frac{Q(3,-3,0)}{4\pi(18)^{3/2}} = \frac{6Q\,\bar{a_x}}{4\pi(18)^{3/2}}$$

$\boxed{\bar{D} = \bar{D_L}+\bar{D_p} = 0}$

As per the given problem $\frac{6Q}{4\pi(18)^{3/2}}\bar{a_x} + 0.64\,\bar{a_x}\times10^{-6} = 0$

$$0.64\times10^{-6} + \frac{6Q}{4\pi(18)^{3/2}} = 0$$

$$Q = \frac{-0.64\times10^{-6}\times4\pi\times(18)^{3/2}}{6} = -102.31\times10^{-6}$$

$Q = -102.31\times10^{-6} = -102.31\,\mu C$

Problem

A charge distribution with spherical symmetry has density

$$\rho_v = \begin{cases} \rho_0 r/R & 0 \le r \le R \\ 0 & r > R \end{cases}$$

Solution:

a) $r < R$ $\quad \Psi = DS = Q_{enc} = \int \rho_v \, dv$

$$\varepsilon_0 E(4\pi r^2) = Q_{enclosed} = \int_0^r \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \rho_v \, r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \int_0^r \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{\rho_0 r}{R} r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \frac{\rho_0}{R} \int_{r=0}^{r} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^3 \sin\theta \, dr\, d\theta\, d\phi$$

$$\varepsilon_0 E(4\pi r^2) = \frac{\rho_0}{R} \cdot \frac{r^4}{4} \cdot 2 \cdot 2\pi = \frac{\rho_0 \pi r^4}{R}$$

$$E = \frac{\rho_0 \pi r^4}{R \cdot \varepsilon_0 \cdot 4\pi r^2} = \frac{\rho_0 r^2}{4\varepsilon_0 R} \qquad \boxed{\bar{E} = \frac{\rho_0 r^2}{4\varepsilon_0 R} \bar{a}_r}$$

b) $r > R$ $\quad \Psi = DS = Q_{enc} = \int \rho_v \, dv$

$$\varepsilon_0 E(4\pi r^2) = Q_{enclosed} = \int_{r=0}^{r} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho_v r^2 \sin\theta \, dr\, d\theta\, d\phi \qquad \boxed{\rho_v = 0 \quad r > R}$$

$$= \int_{r=0}^{R} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\rho_0 r}{R} r^2 \sin\theta \, dr\, d\theta\, d\phi + \int_R^r \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} 0 \cdot r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \frac{\rho_0}{R} \int_{r=0}^{R} r^3 dr \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi = \frac{\rho_0}{R} \cdot \frac{R^4}{4} \cdot 2 \cdot 2\pi = \pi \rho_0 R^3$$

$$E = \frac{\pi \rho_0 R^3}{\varepsilon_0 \cdot 4\pi r^2} = \frac{\rho_0 R^3}{4\varepsilon_0 r^2} \qquad \boxed{\bar{E} = \frac{\rho_0 R^3}{4\varepsilon_0 r^2} \bar{a}_r}$$

Problem: A charge distribution in free space has $\rho_v = 2r \; nC/m^3$ for $0 \le r \le 10$ and zero otherwise. Determine E at $r = 2m$ and $r = 12m$

$\frac{1}{2\varepsilon_0} = 18\pi \times 10^9$

Solution: $\quad Q = \Psi = DS = \int \rho_v \, dv$

$$D(4\pi r^2) = \iiint 2r \cdot r^2 \sin\theta \, dr\, d\theta\, d\phi = 2 \cdot \frac{r^4}{4} \cdot 2 \cdot 2\pi \times 10^{-9}$$

$$D = \frac{2\pi r^4}{4\pi r^2} \times 10^{-9} \Rightarrow \bar{E} = \frac{r^2 \times 10^{-9}}{2\varepsilon_0} \Rightarrow \bar{E} = \frac{r^2 \times 10^{-9}}{2\varepsilon_0} \bar{a}_r$$

$$\bar{E}_{at\, r=2} = \frac{(2)^2 \times 10^{-9}}{2} \times 18\pi \times 10^9 \, \bar{a}_r = 72\pi \bar{a}_r = 226 \bar{a}_r \; V/m$$

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For $r \le 10$,

$$D(4\pi r^2) = \int_0^{r_0}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 2r \cdot r^2 \sin\theta \, dr\, d\theta\, d\phi \times 10^{-9} = 2\left[\frac{r^4}{4}\right]_0^{r_0}\left[-\cos\theta\right]_0^{\pi}\left[\phi\right]_0^{2\pi} \times 10^{-9}$$

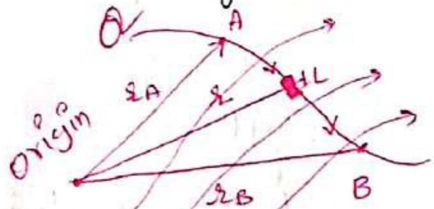
$$D(4\pi r^2) = 2 \cdot \frac{r_0^4}{4} \cdot 2 \cdot 2\pi \times 10^{-9} \quad 2\pi r_0^4 \qquad r_0 = 10\,m$$

$$D = \frac{2\pi r_0^4}{4\pi r^2} \times 10^{-9} = \frac{r_0^4}{2r^2} \times 10^{-9} \Rightarrow \bar{E} = \frac{r_0^4}{2\epsilon_0 r^2} \times 10^{-9}\, \bar{a}_r \qquad r = 12\,m$$

$$\bar{E} = 10^4 \times 10^{-9} \times 15\pi \times 10^9 \times (12)^2 = 1250\pi\, \bar{a}_r$$

$$\boxed{\bar{E} = 3.927\, \bar{a}_r \ \text{kV/m}}$$

Electric potential: Electric potential at point 'p' due to a fixed charge Q is defined as the workdone in moving one coulomb of charge from infinity (∞) to the point P in an electric field against the force created by the fixed charge.



Suppose we wish to move a point charge Q from point A to point B in an electric field E as shown in Figure.

Displacement of point charge Q in electrostatic field E

From Coulomb's law, the force on Q is $F = EQ$ so that the work done in displacing the charge by dL is

$$\boxed{dw = -F \cdot dl = -Q\, E \cdot dl}$$

The negative sign indicates that the work is being done by an external agent. Thus the total workdone, or the potential energy required, in moving Q from A to B is

$$\boxed{W = -Q\int_A^B E \cdot dL \Rightarrow \frac{W}{Q} = -\int_A^B E \cdot dL}$$

Dividing W by Q gives the potential energy per unit charge. This quantity denoted by V_{AB}, is known as the potential difference between points A and B.

Note that

1. In determining V_{AB}, A is the initial point while B is the final point.
2. If V_{AB} is negative, there is a loss in potential energy in moving Q from A to B. this implies that the work is being done by the field. However, if V_{AB} is positive, there is a gain in potential energy in the movement; an external agent performs the work.
3. V_{AB} is independent of the path taken
4. V_{AB} is measured in Joules per Coulomb, commonly referred to as volts (V)

$\bar{E}$ (Electric field) to a point charge Q located at the origin

$$E = \frac{Q}{4\pi\epsilon r^2}\bar{a_r}$$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon r^2}\bar{a_r} \cdot dr\bar{a_r} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r}\right]_{r_A}^{r_B}$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$\boxed{V_{AB} = V_B - V_A}$$

V_A, V_B are the potentials (Absolute potentials) at A and B, respectively.

Now the potential any point due to point charge Q located at the origin and distance is 'r'

$$\boxed{V = \frac{Q}{4\pi\epsilon r} \quad J/c \text{ or volts}}$$

The potential at any point is the potential difference between that point and a chosen point at which the potential is zero.

The potential around a closed path is equal to zero $\boxed{\oint \bar{E} \cdot d\bar{L} = 0}$

→ If any other point is chosen as the reference point, the point at point 'p' is $V = \frac{Q}{4\pi\epsilon r} + c$ where is c is constant can be determined from the potential at the reference point.

Problem: Two point charges $-4\mu C$ and $5\mu C$ are located at $(2,-1,3)$ and $(0,4,-2)$ respectively. Find the potential at $(1,0,1)$ assuming zero potential at infinity.

Solution: $Q_1 = -4\mu C$ $(2,-1,3)$ $\quad Q_2 = 5\mu C$ $(0,4,-2)$

$$V = \frac{Q}{4\pi\varepsilon R} \Rightarrow V = V_1 + V_2 = \frac{Q_1}{4\pi\varepsilon R_1} + \frac{Q_2}{4\pi\varepsilon R_2} + C$$

$$R_1 = |(1,0,1)-(2,-1,3)| = |(-1,1,-2)| = \sqrt{(-1)^2+(1)^2+(-2)^2} = \sqrt{6}$$

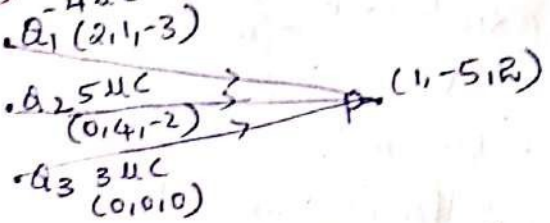
$$= \quad R_2 = |(1,0,1)-(0,4,-2)| = |(1,-4,3)| = \sqrt{(1)^2+(-4)^2+(3)^2} = \sqrt{26}$$

$$V \text{ at } (1,0,1) = -4\times10^{-6}\times9\times10^{9}\left[\left(\frac{1}{\sqrt{6}}\right)\right] + 5\times10^{-6}\times9\times10^{9}\times\frac{1}{\sqrt{26}}$$

$$= 9\times10^{3}\left[\frac{-4}{\sqrt{6}} + \frac{5}{\sqrt{26}}\right] = -5.872\times10^{3} = \underline{-5.872\,KV}$$

Problem: If point charge $3\mu C$ is located at the origin in addition to the two charges of above example problem. Find the potential at $(1,-5,2)$ assuming $V(\infty)=0$

Solution:



$$V = V_1 + V_2 + V_3 = \frac{Q_1}{4\pi\varepsilon R_1} + \frac{Q_2}{4\pi\varepsilon R_2} + \frac{Q_3}{4\pi\varepsilon R_3} + C$$

$$R_1 = |(1,-5,2)-(2,1,-3)| = |(-1,-6,5)| = \sqrt{(-1)^2+(-6)^2+(5)^2} = \sqrt{62}$$

$$R_2 = |(1,-5,2)-(0,4,-2)| = |(1,-9,4)| = \sqrt{1^2+(-9)^2+(4)^2} = \sqrt{98}$$

$$R_3 = |(1,-5,2)-(0,0,0)| = |(1,-5,2)| = \sqrt{1^2+(-5)^2+(2)^2} = \sqrt{30}$$

$$= -4\times10^{-6}\times9\times10^{9}\times\frac{1}{\sqrt{62}} + 5\times10^{-6}\times9\times10^{9}\times\frac{1}{\sqrt{98}} + 3\times10^{-6}\times9\times10^{9}\times\frac{1}{\sqrt{30}}$$

$$= 9\times10^{3}\left[\frac{-4}{\sqrt{62}} + \frac{5}{\sqrt{98}} + \frac{3}{\sqrt{30}}\right] = 9\times10^{3}\times1.560 = \underline{14.04\times10^{3} = 14.04\,KV}$$

Problem: A point charge of $5\,nC$ is located at the origin. If $V = 2V$ at $(0,6,-8)$ find a) The potential at $A(-3,2,6)$ b) The potential at $B(1,5,7)$ c) The potential difference V_{AB}.

Solution:

$$V = \frac{Q}{4\pi\varepsilon_0 R} + C$$

If $V(0,6,-8)$ at is $2V$

$$2 = \frac{5\times10^{-9}\times9\times10^{9}}{10} + c \qquad R = |(0,6,-8)-(0,0,0)| = |(0,6,-8)|$$

$$R = \sqrt{0^2+6^2+(-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$$

$$2 = 4.5 + c$$

$$\underline{c = -2.5}$$

a) The potential at $A(-3,2,6)$

$$V_A = \frac{Q}{4\pi\epsilon R_A} + c \qquad R_A = |(-3,2,6)-(0,0,0)| = |(-3,2,6)|$$

$$R_A = \sqrt{-3^2+2^2+6^2} = \sqrt{49} = 7$$

$$V_A = \frac{5\times10^{-9}\times9\times10^{9}}{7} + (-2.5) = \frac{45}{7} - 2.5 = \underline{3.929 \text{ volts}}$$

b) The potential at $B(1,5,7)$

$$V_B = \frac{Q}{4\pi\epsilon R_B} + c \qquad R_B = |(1,5,7)-(0,0,0)| = |(1,5,7)|$$

$$R_B = \sqrt{1^2+5^2+7^2} = \sqrt{75}$$

$$V_B = \frac{5\times10^{-9}\times9\times10^{9}}{\sqrt{75}} - 2.5 = \frac{45}{\sqrt{75}} - 2.5$$

$$= 5.196 - 2.5 = \underline{2.696 \text{ volts}}$$

c) The potential difference $V_{AB} = V_B - V_A = 2.696 - 3.929$

$$\underline{= -1.233 \text{ volts}}$$

Problem: Calculate the work done in moving a $40\mu C$ charge from a point $A(3, 45^\circ, 135^\circ)$ to $B(4, 120^\circ, 75^\circ)$ in a field having potential given by $V = 25r\cos^2\theta\sin\phi$ volts.

~~solution~~ Solution:

$$W = +Q V_{AB}$$

$$V_A = 25r\cos^2\theta\sin\phi \text{ at } (3, 45^\circ, 135^\circ)$$

$$r = 3,\ \theta = 45^\circ,\ \phi = 135^\circ$$

$$= 25\times3\times\cos^2 45\times\sin 135 = 26.516 \text{ volts}$$

$$V_B = 25r\cos^2\theta\sin\phi \text{ at } (4, 120^\circ, 75^\circ)$$

$$r = 4,\ \theta = 120^\circ,\ \phi = 75^\circ$$

$$= 25\times4\times\cos^2 120\times\sin 75 = 24.148 \text{ volts}$$

$$W = +QV_{AB} = -Q(V_B - V_A) = 40\times10^{-6}[24.148 - 26.516]$$

$$= -94.714\times10^{-6} = \underline{-94.714\,\mu J}$$

The $-ve$ sign indicates the work is being done by the field.

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Relation between E and V:

We know that in a given electric field $\vec{E}$, the potential difference between points A and B is independent of the path chosen i.e.,

$$V_{AB} = -\int_A^B \bar{E}\cdot d\bar{L} \quad \text{and} \quad V_{BA} = -\int_B^A \bar{E}\cdot d\bar{L} = -V_{AB}$$

$$V_{AB} + V_{BA} = 0 = \oint \bar{E}\cdot d\bar{L} = 0$$

Thus the potential difference around a closed path must be zero.

$$\oint \bar{E}\cdot dL = \int (\nabla \times \bar{E})\cdot d\vec{S} = 0$$

$$\nabla \times \vec{E} = 0$$

A Conservative field (also called path independent vector field) is a vector field whose line integral over any curve depends only on the endpoint of curve and it is independent of the path.

The curl of the electric field must be zero.

Any electric field that satisfies the above equation is called a conservative field or an irrotational one.

$$V = -\int \bar{E}\cdot d\bar{L} = -\int dV \qquad \bar{E} = E_x \bar{a_x} + E_y \bar{a_y} + E_z \bar{a_z}$$

$$d\bar{L} = dx\,\bar{a_x} + dy\,\bar{a_y} + dz\,\bar{a_z}$$

$$dV = -\bar{E}\cdot d\bar{L} = -E_x dx - E_y dy - E_z dz$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$\boxed{\nabla \times \vec{E} = 0,\ \text{Maxwell's second equation}}$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = \frac{-\partial V}{\partial z}$$

$$\bar{E} = E_x \bar{a_x} + E_y \bar{a_y} + E_z \bar{a_z}$$

$$\bar{E} = -\frac{\partial V}{\partial x}\bar{a_x} + \frac{\partial V}{\partial y}\bar{a_y} - \frac{\partial V}{\partial z}\bar{a_z}$$

$$= -\left[\frac{\partial}{\partial x}\bar{a_x} + \frac{\partial}{\partial y}\bar{a_y} + \frac{\partial}{\partial z}\bar{a_z}\right] V$$

$$\boxed{\bar{E} = -\nabla V}$$

Thus the electric field intensity is equal to the gradient of the potential. The negative sign indicates that the field direction is opposite to the direction of the potential increment.

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Problem: Define Conservative field. Find whether the electrostatic field given by $\bar{E} = yz\bar{x} + xz\bar{y} + xy\bar{z}$ V/m is Conservative or not.

$\boxed{\text{Dec-2015}}$

Solution:

$$\nabla \times \bar{E} = \begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = \bar{x}[x-x] + \bar{y}[y-y] + \bar{z}[z-z] = 0$$

$\nabla \times \bar{E} = 0$, so the field is Conservative field.

Problem: Given $V = 5x^3y^2z$ and $\varepsilon = 2.25\epsilon_0$ find i) E at point $P(-3,1,2)$ ii) ρ_v at P

$\boxed{\text{May-2015}}$

Solution:

i) $\bar{E} = -\nabla V$

$$= -\left[\frac{\partial}{\partial x}\bar{a_x} + \frac{\partial}{\partial y}\bar{a_y} + \frac{\partial}{\partial z}\bar{a_z}\right](5x^3y^2z)$$

$$= -\frac{\partial}{\partial x}(5x^3y^2z)\bar{a_x} - \frac{\partial}{\partial y}(5x^3y^2z)\bar{a_y} - \frac{\partial}{\partial z}(5x^3y^2z)\bar{a_z}$$

$$\bar{E} = -15x^2y^2z\bar{a_x} - 10x^3yz\bar{a_y} - 5x^3y^2\bar{a_z} \quad \text{at } P(-3,1,2)$$

$x=-3, y=1, z=2$

$$= -15(-3)^2(1)^2(2)\bar{a_x} - 10(-3)^3(1)(2)\bar{a_y} - 5(-3)^3(1)^2\bar{a_z}$$

$$= -270\bar{a_x} + 540\bar{a_y} + 45\bar{a_z} \text{ V/m}$$

ii) $\nabla \cdot \bar{D} = \rho_v$

$$\rho_v = \nabla \cdot \varepsilon\bar{E} = 2.25\epsilon_0(\nabla \cdot \bar{E})$$

$$= 2.25 \times 8.856 \times 10^{-12}\left(\frac{\partial}{\partial x}\bar{a_x} + \frac{\partial}{\partial y}\bar{a_y} + \frac{\partial}{\partial z}\bar{a_z}\right)\cdot\left(-15x^2y^2z\bar{a_x} - 10x^3yz\bar{a_y} - 5x^3y^2\bar{a_z}\right)$$

$$= 2.25 \times 8.856 \times 10^{-12}\left[-30xy^2z\,\bar{a_x} - 10x^3z\,\bar{a_y}\right]$$

at $P(-3,1,2)$

$x=-3, y=1, z=2$

$$= 2.25 \times 8.856 \times 10^{-12}\left[-30(-3)(1)^2(2)\bar{a_x} - 10(-3)^3(2)\bar{a_y}\right]$$

$$= 19.926 \times 10^{-12}\left[180\bar{a_x} + 540\bar{a_y}\right]$$

$$= (3.586\bar{a_x} + 10.760\bar{a_y}) \times 10^{-9}$$

$$= 3.586\bar{a_x} + 10.760\bar{a_y} \text{ nC/m}^3$$

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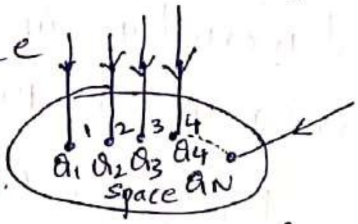
Energy density in an Electrostatic field:
Energy stored in an Electrostatic field:

The energy stored in an electric field due to N point charges is expressed as

$$\boxed{W_E = \frac{1}{2}\sum_{i=1}^{N} V_i Q_i \text{ Joules}}$$

proof: Consider a free space where there is no electric field. Let a point charge Q_1 be moved from infinity to point 1 in the free space as shown in figure



pointcharges in space.

Since there is no field, the work done is zero i.e. $W_1 = 0$
Now due to Q_1 in the free space, an electric field is established.

If a point charge Q_2 is moved from infinity to point 2 in this field, then the work done in keeping Q_2 is

$$\boxed{W_2 = V_{21} Q_2}$$

where V_{21} is the potential at point 2 due to Q_1. Now the field increases due to Q_1 and Q_2.

Where V_{21} is the potential at point 2 due to Q_1. Now the fied increases due to Q_1 and Q_2.

If another point charge Q_3 is moved from infinity to a point 3 in this field, then the work done in keeping Q_3 is

$$\boxed{W_3 = V_{31} Q_3 + V_{32} Q_3}$$

Where V_{31} is the potential at point 3 due to Q_1. and V_{32} is the potential at point 3 due to Q_2.

$$\boxed{W_4 = V_{41} Q_4 + V_{42} Q_4 + V_{43} Q_4}$$

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Similarly, if the point charges $Q_4, Q_5, \cdots Q_{N-1}$ etc are moved into this field, the field strength increases.

Finally, if the point charge Q_N is moved from infinity to a point N in this field, then the work done in keeping Q_N is

$$\boxed{W_E = V_{N1}Q_N + V_{N2}Q_N + \cdots + V_{NN-1}Q_N}$$

The total work done in positioning all the point charges at their respective points is

$$W_E = W_1 + W_2 + W_3 + \cdots + W_N$$

$$= 0 + (V_{21}Q_2) + (V_{31}Q_3 + V_{32}Q_3) + (V_{41}Q_4 + V_{42}Q_4 + V_{43}Q_4)$$
$$+ (V_{51}Q_5 + V_{52}Q_5 + V_{53}Q_5 + V_{54}Q_5 + \cdots$$
$$+ (V_{N1}Q_N + V_{N2}Q_N + \cdots + V_{NN-1}Q_N) \quad \text{—— (1)}$$

The total work done in positioning all the point charges in a space is called the potential energy or the energy stored in an electrostatic field.

To simplify the equation, let us consider that all the point charges are placed in reverse order. That is

$$\boxed{Q_i V_{ij} = Q_j V_{ji}}$$

$$W_E = (V_{12}Q_1) + (V_{13}Q_1 + V_{23}Q_2) + (V_{14}Q_1 + V_{24}Q_2 + V_{34}Q_3)$$
$$+ (V_{15}Q_1 + V_{25}Q_2 + V_{35}Q_3 + V_{45}Q_4) + \cdots$$
$$+ (V_{1N}Q_1 + V_{2N}Q_2 + V_{3N}Q_3 + \cdots + V_{N-1,N}Q_N) \quad \text{—— (2)}$$

Equation-(1) + Equation-2

$$2W_E = Q_1(V_{12} + V_{13} + V_{14} + V_{15} + \cdots + V_{1N}) +$$
$$Q_2(V_{21} + V_{23} + V_{24} + V_{25} + \quad + V_{2N}) +$$
$$\vdots$$
$$Q_N(V_{N1} + V_{N2} + V_{N3} + \cdots V_{NN}).$$

Let the potential at Q_1 due to all the other charges be

potential at Q_1 be $V_1 = V_{12} + V_{13} + V_{14} + \cdots + V_{1N}$

Q_2 be $V_2 = V_{21} + V_{23} + V_{24} + \cdots + V_{2N}$

and Q_N $= V_{N1} + V_{N2} + V_{N3} + \cdots + V_{NN}$

$$2W_E = Q_1V_1 + Q_2V_2 + Q_3V_3 + \cdots + Q_NV_N$$

$$W_E = \frac{1}{2}(Q_1V_1 + Q_2V_2 + \cdots + Q_NV_N)$$

$$\boxed{W_E = \frac{1}{2}\sum_{i=1}^{N} Q_N V_N \text{ Joules.}}$$

Similarly, for ~~a line of charge density~~ ρ_L Charge densities

$$\boxed{W_E = \frac{1}{2}\int V \rho_L \, dL \text{ Joules}}$$ Due to line chargedensity

$$\boxed{W_E = \frac{1}{2}\int V \rho_s \, ds \text{ Joules}}$$ Due to surface chargedensity

$$\boxed{W_E = \frac{1}{2}\int V \rho_v \, dv \text{ Joules}}$$ Due to volume chargedensity

Enery Stored in Terms of $\vec{E}$:

$$W_E = \frac{1}{2}\int V \rho_v \, dv \text{ Joules} \qquad \nabla \cdot \bar{D} = \rho_v$$

$$W_E = \frac{\varepsilon}{2}\int V (\nabla \cdot \bar{E}) \, dv$$

$$\boxed{(\nabla \cdot \bar{E})V = \nabla \cdot V\bar{E} - \bar{E} \cdot \nabla V}$$ Vector identity

$$W_E = \frac{\varepsilon}{2}\left[\int_V (\nabla \cdot V\bar{E}) dv - \int_V \bar{E} \cdot \nabla V dv\right]$$

using divergence theorem $\boxed{\int_V (\nabla \cdot V\bar{E}) dv = \oint_S V\bar{E} \cdot d\bar{s} = 0}$

no closed surface encloses $\bar{E}$

$$W_E = \frac{-\varepsilon}{2}\int \bar{E}\cdot\nabla V\, dv$$

$$\nabla V = -\bar{E}$$

$$= \frac{-\varepsilon}{2}\int \bar{E}\cdot(-\bar{E})\, dv = \frac{\varepsilon}{2}\int E^2 dv$$

$$W_E = \frac{1}{2}\int \varepsilon E^2 dv = \frac{1}{2}\int \bar{D}\cdot\bar{E}\, dv \text{ Joules}$$

Therefore the Energy density in an Electrostatic field is

$$\boxed{\text{Energy Density } \omega_E = \frac{W_E}{V} = \frac{1}{2}\varepsilon E^2 \text{ Joules}/m^3}$$

Energy in a capacitor:

Assume that a Capacitor C is charged to a voltage, V.

$$\cancel{W_C = \frac{1}{2}\int \bar{D}\cdot\bar{E}\, dv}$$

If the potential difference across the plates at any instant of charging is V, this is equal to the work done in shifting one coulomb of charge ~~on~~ from one plate to another. If dQ is the charge transferred, workdone is

$$dW_C = V\,dQ \qquad Q = CV$$

$$dQ = C\,dV$$

$$dW_C = CV\,dV$$

Total workdone in producing a potential of V is

$$W_C = \int_0^V CV\, dv = C\left[\frac{V^2}{2}\right]_0^V = \frac{1}{2}CV^2$$

$$\boxed{W_C = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C} \text{ Joules}}$$

Problem: Three point charges $-1nc$, $4nc$ and $3nc$ are located at $(0,0,0)$, $(0,0,1)$ and $(1,0,0)$ respectively. Find the energy in the system. $\boxed{\text{May-2017}}$

Solution: $W = W_1 + W_2 + W_3$ $\qquad$ V_{21} — potential at point 2 due to Q_1

$$= 0 + Q_2 V_{21} + Q_3 (V_{31} + V_{32})$$

$$= Q_2 \cdot \frac{Q_1}{4\pi\varepsilon_0 |(0,0,1)-(0,0,0)|} + \frac{Q_3}{4\pi\varepsilon_0}\left[\frac{Q_1}{|(1,0,0)-(0,0,0)|} + \frac{Q_2}{|(1,0,0)-(0,0,1)|}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[Q_1Q_2 + Q_1Q_3 + \frac{Q_2Q_3}{\sqrt{2}}\right] \qquad \frac{1}{4\pi\varepsilon_0} = 9\times10^{9}$$

$$= 9\times10^{9}\left[-1\times10^{-9}\times4\times10^{-9} + (-1\times10^{-9}\times3\times10^{-9}) + (4\times10^{-9}\times3\times10^{-9})\right]$$

$$= 9\times10^{9}\times10^{-18}\left[-4-3+\frac{12}{\sqrt{2}}\right] = 13.37\times10^{-9} = 13.37\ n\text{Joules}.$$

Problem: Point charges $Q_1 = 1nc$, $Q_2 = -2nc$, $Q_3 = 3nc$ and $Q_4 = -4nc$ are postioned one at a time in that order at $(0,0,0)$, $(1,0,0)$ $(0,0,-1)$ and $(0,0,1)$ respectively. Calculate the energy in the system after each charge is positioned. $\quad \frac{1}{4\pi\varepsilon_0} = 9\times10^{9}$

Solution:

After Q_1 is energy $W_{Q_1} = 0$

After Q_2 is energy $W_{Q_2} = Q_2 V_{21} = \frac{Q_2\ Q_1}{4\pi\varepsilon_0 |(1,0,0)-(0,0,0)|}$

$$= -2\times10^{-9}\times1\times10^{-9}\times9\times10^{9}\times1 = -18\times10^{-9}\ \text{Joules}$$

After Q_3 is energy $W_{Q_3} = \cancel{Q} + Q_2V_{21} + Q_3(V_{31}+V_{32})$

$$= \frac{Q_2 Q_1}{4\pi\varepsilon_0 |(1,0,0)-(0,0,0)|} + \frac{Q_3}{4\pi\varepsilon_0}\left[\frac{Q_1}{|(0,0,-1)-(0,0,0)|} + \frac{Q_2}{|(0,0,-1)-(1,0,0)|}\right]$$

$$= -2\times10^{-9}\times1\times10^{-9}\times9\times10^{9} + 3\times10^{-9}\times9\times10^{+9}\left[10^{-9} - \frac{2\times10^{-9}}{\sqrt{2}}\right]$$

$$= -18\times10^{-9} + 27\left[1-\frac{2}{\sqrt{2}}\right]\times10^{-9}$$

$$= -29.18\times10^{-9}\ \text{Joules}.$$

After Q_4 energy is $W_{Q_4} = Q_4(V_{41}+V_{42}+V_{43}) + \underbrace{Q_3(V_{31}+V_{32}) + Q_2V_{21}}_{-29.18\times10^{-9}}$

$$= -Q_4\left(\frac{Q_1}{4\pi\varepsilon_0|(0,0,1)-(0,0,0)|} + \frac{Q_2}{4\pi\varepsilon_0|(0,0,1)-(1,0,0)|} + \frac{Q_3}{4\pi\varepsilon_0|(0,0,1)-(0,0,-1)|}\right) - 29.18\times10^{-9}$$

$$= -4\times10^{-9}\times9\times10^{9}\left(\frac{1\times10^{-9}}{1} - \frac{2}{\sqrt{2}} + \frac{3}{2}\right) - 29.18\times10^{-9}$$

$$= -36\times10^{-9}\left(1-\frac{2}{\sqrt{2}}+\frac{3}{2}\right) - 29.18\times10^{-9} = -68.27\times10^{-9}$$

$$= -68.27 \text{ n joules}$$

Problem: Find the workdone in carrying a 5-c charge from $P(1,2,-4)$ to $R(3,-5,6)$ in an electric field

$\bar{E} = a_x + x^2 a_y + 2yz a_z$ V/m $\boxed{\text{Dec - 2017}}$

solution:

Breakup paths

$P(1,2,-4) \xrightarrow{} R(3,-5,6)$

$P'(3,2,-4) \rightarrow R'(3,-5,-4)$

$$\frac{-W}{Q} = -\int \bar{E}\cdot d\bar{L} =$$

$$= \int_P^{P'}\int_{P'}^{R'}\int_{R'}^{R} (a_x + x^2a_y + 2yz\,a_z)\cdot(dx\,a_x + dy\,a_y + dz\,a_z)$$

$$= \cancel{\int (dx + x^2dy + 2yz\,dz)}$$

$$= \int dx$$

$$= \int_{x=1}^{3} dx + \int_{y=2}^{-5} x^2dy + \int_{z=-4}^{6} 2yz\,dz \qquad (z=-4) \quad (y=-5)$$

$$= (x)_1^3 + (-4)^2(y)_2^{-5} + 2(-5)\left(\frac{z^2}{2}\right)_{-4}^{6}$$

$$= 2 + 16(-7) + 2(-5)\left(\frac{36-16}{2}\right) = 2-112-100 = -210$$

$$W = -210(-Q) = 210(5) = 1050\text{ J}$$

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1) Del operator (∇):

The del operator, written ∇, is the vector differential operator.

In cartesian coordinate system (x, y, z)

a) $$\boxed{\nabla = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z}$$

b) In cylindrical coordinate system (ρ, ϕ, z)

$$\boxed{\nabla = \frac{\partial}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\bar{a}_\phi + \frac{\partial}{\partial z}\bar{a}_z}$$

c) In spherical coordinate system (r, θ, ϕ)

$$\boxed{\nabla = \frac{\partial}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\bar{a}_\phi}$$

2) Gradient of a scalar:

The gradient of a scalar field V is a vector that represents both magnitude and the direction of the maximum space rate of increase of V.

a) $$\boxed{\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z}$$ (Cartesian Coordinate system)

b) In cylindrical Coordinate system (ρ, ϕ, z)

$$\boxed{\nabla V = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z}$$

c) In spherical coordinate system (r, ϕ, z)

$$\boxed{\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi}$$

3) Divergence of a Vector and Divergence theorem

The divergence of $\bar{A}$ at a given point 'p' is the outward flux per unit volume as the volume shrinks about 'p'.

$$\bar{A} = A_x \bar{a_x} + A_y \bar{a_y} + A_z \bar{a_x}$$

a) In cartesian Coordinate system (x, y, z)

$$\boxed{\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

b) In cylindrical coordinate system (ρ, ϕ, z)

$$\bar{A} = A_\rho \bar{a_\rho} + A_\phi \bar{a_\phi} + A_z \bar{a_z}$$

$$\boxed{\nabla \cdot \bar{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}}$$

c) In spherical coordinate system (r, θ, ϕ)

$$\bar{A} = A_r \bar{a_r} + A_\theta \bar{a_\theta} + A_\phi \bar{a_\phi}$$

$$\boxed{\nabla \cdot \bar{A} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta} \frac{\partial A_\phi}{\partial \phi}}$$

The divergen theorem states that the total outward flux of a vector field $\bar{A}$ through the closed surface 'S' is the same as the volume integral of the divergence of A. It is also known as Gauss- Ostrogradsky theorem. $\boxed{\oint \bar{A} \cdot d\bar{s} = \int_V \nabla \cdot \bar{A} \, dv}$

4) Curl of a vector and Stoke's theorem:

The curl of A is an axial (or rotational) vector whose magnitude is the maximum circulation of A per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented so as to make the circulation maximum.

a) In Cartesian Coordinate system (x, y, z)

$$\bar{A} = A_x \bar{a_x} + A_y \bar{a_y} + A_z \bar{a_x}$$

$$\nabla \times \bar{A} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\boxed{\nabla \times \bar{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\bar{a_x} - \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\bar{a_y} + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\bar{a_z}}$$

b) In cylindrical coordinate system (ρ, ϕ, z)

$$\bar{A} = A_\rho \bar{a}_\rho + A_\phi \bar{a}_\phi + A_z \bar{a}_z$$

$$\nabla \times \bar{A} = \frac{1}{\rho} \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\boxed{\nabla \times \bar{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\bar{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\bar{a}_\phi + \frac{1}{\rho}\left[\frac{\partial (\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]\bar{a}_z}$$

c) In spherical coordinate system (r, θ, ϕ)

$$\bar{A} = A_r \bar{a}_r + A_\theta \bar{a}_\theta + A_\phi \bar{a}_\phi$$

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r a_\theta & r\sin\theta \, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r\sin\theta \, A_\phi \end{vmatrix}$$

$$\boxed{\begin{aligned} \nabla \times \bar{A} = \; & \frac{1}{r\sin\theta}\left[\frac{\partial (A_\phi \sin\theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\bar{a}_r \; + \\ & \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial r}\right]\bar{a}_\theta \; + \\ & \frac{1}{r}\left[\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\bar{a}_\phi \end{aligned}}$$

Stokes theorem states that the circulation of a vector field A around a (closed) path L is equal to the surface integral of the curl of A over the open surface S bounded by L provided that $\bar{A}$ and $\nabla \times \bar{A}$ are continuous on S

$$\boxed{\oint \bar{A} \cdot d\bar{L} = \int_S (\nabla \times A) \cdot d\bar{S}}$$

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Laplacian of a scalar:

The Laplacian of a scalar field V, written as $\nabla^2 V$, is the divergence of the gradient of V.

$$\nabla \cdot \nabla V = \nabla^2 V = \text{Laplacian } V$$

a) In cartesian coordinate system (x, y, z)

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}}$$

b) In cylindrical coordinate system (ρ, ϕ, z)

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}}$$

c) In spherical coordinate system (r, θ, ϕ)

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}}$$

properties of Conductors:

1. Charge density is zero
2. In static Conductors, current flow is zero.
3. Conductivity is very large.
4. Resistivity is small
5. Good Conductors reflect electric and magnetic fields Completely
6. A conductor is an equipotential body
7. $E = -\nabla V = 0$ in a conductor., $\rho_v = 0$, $V_{ab} = 0$ inside a conductor

Electric Current:

The current through a given medium is defined as charge passing through the medium per unit time. It is a scalar,

that is $\quad I = \frac{dq}{dt}$ Ampere & Coulomb / sec

Current is of three types:

1. Convection Current
2. Conduction Current
3. Displacement Current.

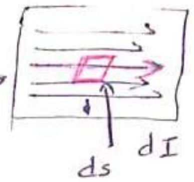
1. Convection current: It is defined as the current produced by a beam of electrons flowing through an insulating medium. This does not obey ohm's law. Ex: Current through vacuum, liquid and so on is Convection Current.

2. Conduction current: It is defined as the current produced due to flow of electrons in a Conductor. This obeys ohm's law. For Example: current in a Conductor like copper is Conduction current.

3. Displacement current: It is defined as the current which flow as a result of time-varying electric field in a dielectric material. For example current through a capacitor when a time-varying voltage is applied is displacement current.

Current density: The current density at a given point is the current through a unit normal area at that point. It is a vector. It is represented by J. units are A/m^2

$I =$



$$J = \frac{dI}{ds}\bar{a}_n \Rightarrow I = \oint \bar{J}\cdot\bar{ds}$$

1. Convection current density ($\bar{J}$) It is defined as the convection current at a given point through a unit normal area at that point, that is convection current density.

2. Conduction current density ($\bar{J}_c$): It is defined as the conduction current at a given point through a unit normal area at that point

$$\bar{J}_c = \sigma\bar{E} = \frac{dI}{ds}\bar{a}_n$$

→ point form of ohm's law

3. Displacement current density ($\bar{J}_d$): It is defined as the rate of displacement electric flux density with time, that is

$\bar{D} = \varepsilon\bar{E}$

$$\bar{J}_d = \frac{\partial\bar{D}}{\partial t} = \frac{\varepsilon\,\partial\bar{E}}{\partial t}$$

$$\bar{J} = \bar{J}_c + \bar{J}_d$$

Problem: If $\bar{J} = \frac{1}{r^3}(2\cos\theta\, \bar{a}_r + \sin\theta\, \bar{a}_\theta)$ A/m². Calculate the current passing through

a) A hemispherical shell of radius 20cm

b) A spherical shell of radius 10cm

Solution:

a) $I = \oint_S \bar{J}\cdot d\bar{s}$ $\qquad d\bar{s}_r = r^2 \sin\theta\, d\theta\, d\phi\, \bar{a}_r$

$$= \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \frac{1}{r^3}(2\cos\theta\,\bar{a}_r + \sin\theta\,\bar{a}_\theta)\cdot(r^2\sin\theta\, d\theta\, d\phi\, \bar{a}_r)$$

$$= \frac{2}{r}\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \cos\theta\sin\theta\, d\theta\, d\phi = \frac{2}{r}\int_{\theta=0}^{\pi/2}\frac{\sin 2\theta}{2}d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{\cancel{2}}{r}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2}[\phi]_0^{2\pi} = \frac{1}{0.2}\left[\frac{1+1}{2}\right][2\pi]$$

$$= 10\pi = \underline{31.4 \text{ Ampere}}$$

b) $$I = \oint \bar{J}\cdot d\bar{s} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{1}{r^3}(2\cos\theta\, \bar{a}_r + \sin\theta\,\bar{a}_\theta)\cdot(r^2\sin\theta\, d\theta\, d\phi\, \bar{a}_r)$$

$$= \frac{2}{r}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\cos\theta\sin\theta\, d\theta\, d\phi = \frac{1}{r}\int_{\theta=0}^{\pi}\sin 2\theta\, d\theta\int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{1}{r}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi}[\phi]_0^{2\pi} = \frac{1}{0.1}\left[\frac{-1+1}{2}\right][2\pi - 0] = \underline{0 \text{ Amp}}$$

Problem: For the current density $\bar{J} = 10z\sin^2\phi\, \bar{a}_\rho$ A/m², Find the current through the cylindrical surface $\rho = 2$, $1 \le z \le 5$ m

Solution:

$$d\bar{s}_\rho = \rho\, d\phi\, dz\, \bar{a}_\rho$$

$$I = \oint \bar{J}\cdot d\bar{s} = \int_{\phi=0}^{2\pi}\int_{z=1}^{5} 10z\sin^2\phi\, \bar{a}_\rho\cdot\rho\, d\phi\, dz\, \bar{a}_\rho$$

$$= 10\cdot\rho^2\cdot\int_{\phi=0}^{2\pi}\int_{z=1}^{5}\sin^2\phi\, d\phi\, z\, dz \qquad \int\sin^2\phi = \int\left(\frac{1-\cos 2\phi}{2}\right) = \frac{1}{2}\left[\phi - \frac{\sin 2\phi}{2}\right]$$

$$= 10(2)^2\,(z^2/2)_1^5\cdot\frac{1}{2}\left[\phi - \frac{\sin 2\phi}{2}\right]_0^{2\pi}$$

$$= 10\times\cancel{4}\times\frac{1}{\cancel{2}}\times 24\times\frac{1}{\cancel{2}}[2\pi - 0] = 240\pi = \underline{754 \text{ Amperes}}$$

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Dielectrics: An ideal dielectric material is one which does not contain free electrons.

An ideal dielectric material is one for which there exists a large forbidden gap between valence band and conduction band.

properties:
1. Conductivity is zero
2. volume charge density $\rho_v = 0$
3. Electric and magnetic fields exist in a dielectric material
4. Resistivity is ∞
5. There exists no free electrons.

polar type of dielectrics: If this material kept in an electric field, all the positive charges move in the direction of the electric field and all the negative charges move in the opposite direction. Then the material is said to be under a state of polarization.

Ex: water, hydrochloric acid, sulphur dioxide.

Nonpolar type of dielectrics: If this material is placed in an electric field, the centres of positive and negative charges are displaced and there exists a distance between them, that is, dipole moment is induced. Then the material is said to be under a state of polarization.

Dielectric materials are not polarized in the absence of electric field and they are polarised in the presence of an electric field

Ex: oxygen, hydrogen, nitrogen

Dielectric constant and Dielectric strength:

Dielectric constant (or relative permittivity) ε_r is the ratio of the permittivity of dielectric to that of free space

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 1 + \chi_e \ (\text{chi})$$

ε_r is always greater or equal to unity

For free space and non dielectric materials (such as metals) $\varepsilon_r = 1$

Electric susceptability describes the response of material to electric field. It is represented by χ_e

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No dielectric is ideal.
When the electric field in a dielectric is sufficiently large, it begins to pull electrons completely out of the molecules, and the dielectric becomes Conducting. Dielectric breakdown is said to have occured when dielectric becomes Conducting. Dielectric breakdown occurs in all kinds of dielectric materials (gases, liquids, solids) and depends on the nature of material temperature, humidity and the amount of time that the field is applied. The minimum value of the electric field at which dielectric breakdown occurs is called the dielectric strength of the dielectric material.

The dielectric strength is the maximum electric field that a dielectric can tolerate or withstand without breakdown

S.No.	Material	Dielectric Constant (ϵ_r)	Dielectric strength E (V/m)
1.	Barium titanate	1200	7.5×10^6
2.	paper	7	12×10^6
3.	Glass	5-10	35×10^6
4.	Mica	6	70×10^6
5.	Bakelite	5	20×10^6
6.	Quartz	5	30×10^6
7.	Rubber	3.1	25×10^6
8.	petroleum oil	2.1	12×10^6
9.	Air	1	3×10^6

The time rate of decrease of charge within a given volume must be equal to the net outword current flow through the closed surface of the volume.

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Continuity equation:

$$\nabla\cdot\bar{J} = -\dot{\rho}_v = \frac{-\partial \rho_v}{\partial t}$$

Proof: If Q_i is the charge inside a closed surface, the rate of decrease of charge due to the outward flow of current is given by $\frac{-dQ_i}{dt}$

$$I = -\frac{dQ_i}{dt} = \oint_s \bar{J}\cdot ds$$

* It is derived from the principle of Conservation of charge.

$$\oint \bar{J}\cdot d\bar{s} = \int_v \nabla\cdot\bar{J}\, dv$$

$$\int \nabla\cdot\bar{J}\, dv = -\frac{dQ}{dt} = -\frac{d}{dt}\int \rho_v\, dv = -\int \frac{\partial \rho_v}{\partial t}\, dv$$

$$\boxed{\nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t} = \dot{\rho}_v}$$

for steady currents $\dot{\rho}_v = 0$

* $\nabla\cdot\bar{J} = 0$, showing that the total charge leaving a volume is the same as the total charge entering.

Relaxation Time (Rearrangement time) (T_r)

Relaxation time is the time takes a charge placed in the interior of a material to drop to $e^{-1} = 36.8$ percent of its initial value $T_r = \frac{\varepsilon}{\sigma}$ sec

ε = permittivity = F/m

σ = Conductivity = mho/m

Problem: Find the relaxation time of sea water whose $\varepsilon_r = 81$ and Conductivity $\sigma = 5$ mho/m

Solution:

$$T_r = \frac{\varepsilon}{\sigma} = \frac{81\times 8.856\times10^{-12}}{5} = 143.37\times10^{-12}\ \text{sec}$$

Ex: Conductor copper $\sigma = 5.8\times10^{7}$ mho/m, $\varepsilon_r = 1$ $T_r = \frac{\varepsilon_r\varepsilon_0}{\sigma} = 1.5\times10^{-19}$ s showing rapid decay of charge placed inside copper. This implies that for good conductors, the relaxation time is so short that most of the charge will vanish from any interior point and appear at the surface. For quartz $\sigma = 10^{-17}$ mho/m, $\varepsilon_r = 5$ $T_r = \frac{5\times 8.856\times10^{-12}}{10^{-17}} = 42.93\times10^{5}$ sec $= 51.2$ days

For good dielectrics, one may consider the introduced charge to remain wherever placed.

Linear, Isotropic, and Homogeneous Dielectrics:

A material is said to be linear if D varies linearly with E and nonlinear otherwise.
A material is linear if σ does not vary with E (conductors)
Material for which ε (or σ) does not vary in the region being considered and is therefore the same at all points (i.e. independent of x, y, z) are said to be homogeneous.
For homogeneous σ is the same at all points (conductors)
They are said to be inhomogeneous (or nonhomogeneous) when ε is dependent of the space coordinates.

The atmosphere is a typical example of an inhomogeneous medium, its permittivity varies with altitude.

Materials for which D and E are in the same direction are said to be isotropic. σ does not vary with direction. (conductors)
Isotropic are those which have the same properties in all directions.

For anisotropic (or nonisotropic) materials, D, E and P are not parallel.

example for anisotropic materials are Crystalline materials and magnetized plasma.

A dielectric material (in which $D = \varepsilon E$ applies) is linear if ε does not change with the applied E field, homogeneous if ε does not change from point to point, and isotropic if ε does not change with direction.

For a Conducting material in which $J = \sigma E$ Applies
The material is linear if σ does not vary with E,
homogeneous if σ is same at all points,
and isotropic if σ does not vary with direction.

poisson's and Laplace's Equations:

poisson's and Laplace's equations are easily derived from Gauss's law (for a linear material medium)

$$\nabla \cdot \bar{D} = \rho_v$$

From the defination of electric flux density $\bar{D}$

$$\bar{D} = \varepsilon \bar{E} \quad \Rightarrow \quad \nabla \cdot \varepsilon \bar{E} = \rho_v$$

From the definatin of Electric field Intensity interms of potential $\bar{E} = -\nabla V$

$$\nabla \cdot (\varepsilon(-\nabla V)) = \rho_v$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}}$$ This is Known as poisson's equation

A special case of this equation occurs when $\rho_v = 0$ (charge free region)

$$\boxed{\nabla^2 V = 0}$$ This is Known as Laplace's Equation

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Laplace's equation is of primary importance in solving electrostatic problems involving a set of Conductors maintained at different potentials. Examples of such problems include Capacitors and vacuum tube diodes. Laplace's and poisson's equations are not only useful in solving electrostatic field problem, they are used in various other field problems.

'V' would be interpreted as magnetic potential in magnetostatics as temperature in heat Conduction, as stress function in fluid flow, and as pressure head in seepage.

Problem: If a potential $V = x^2yz + Ay^3z$ a) Find A so that Laplace's equation is satisfied b) with the value of A, determine electric field at (2,1,-1)

Solution: a) Laplace's Equation $\nabla^2 V = 0$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(x^2yz + Ay^3z) = 0$$

$$2yz + 0 + 0 + 6Ayz + 0 + 0 = 0$$

$$2yz + 6Ayz = 0$$

$$\boxed{A = -1/3}$$

b) $V = x^2yz - \frac{1}{3}y^3z$

$$E = -\nabla V = -\left[\frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z\right](x^2yz - \tfrac{1}{3}y^3z)$$

$$= -[2xyz - 0]\bar{a}_x - [x^2z - y^2z]\bar{a}_y - [x^2y - \tfrac{1}{3}y^3]\bar{a}_z$$

E at (2,1,-1) $\quad x=2, y=1, z=-1$

$$\bar{E} = 4\bar{a}_x + 3\bar{a}_y - 3.66\bar{a}_z \text{ V/m or N/C}$$

Problem: In a one-dimensional device, the charge density is given by $\rho_v = \frac{\rho_0 x}{a}$. If $E = 0$ at $x = 0$ and $V = 0$ at $x = a$ find V and E.

Solution:

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\frac{d^2V}{dx^2} = \frac{-\rho_0 x}{a\epsilon}$$

$$\frac{dV}{dx} = -\frac{\rho_0 x^2}{2a\epsilon} + A$$

$$V = -\frac{\rho_0 x^3}{6a\epsilon} + Ax + B$$

$$\bar{E} = -\nabla V = -\left[\frac{\partial}{\partial x}\bar{a}_x\right]V$$

$$= +\left[-\frac{\partial V}{\partial x}\right]\bar{a}_x$$

$$\bar{E} = \left[\frac{\rho_0 x^2}{2\epsilon a} + A\right]\bar{a}_x$$

$$E = |\bar{E}| = \left(\frac{\rho_0 x^2}{2\epsilon a} - A\right)$$

If $\bar{E} = 0$ at $x = 0$ then

$$0 = 0 - A$$

$$\boxed{A = 0}$$

If $V = 0$ at $x = a$ then $0 = \frac{-\rho_0 a^3}{6a\epsilon} + 0 + B$

$$\boxed{B = \frac{\rho_0 a^2}{6\epsilon}}$$

$$\boxed{V = \frac{-\rho_0 x^3}{6\epsilon a} + 0(x) + \frac{\rho_0 a^2}{6\epsilon} = \frac{\rho_0}{6\epsilon a}\left[a^3 - x^3\right]}$$

$$\boxed{\bar{E} = \frac{\rho_0 x^2}{2\epsilon a}\bar{a}_x}$$

Problem: Two infinitely large conducting plates are located at $x = 1$ and $x = 4$. The space between them is free space with charge distribution $\frac{x}{6\pi}$ nc/m³. Find V at $x = 2$ if $V(1) = -50V$ and $V(4) = 50V$

Solution: $\epsilon_0 = \frac{1}{36\pi \times 10^9}$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon_0} \Rightarrow \frac{d^2V}{dx^2} = \frac{-x/6\pi \times 10^{-9}}{10^{-9}/36\pi} = -6x$$

$$\frac{d^2V}{dx^2} = -6x \Rightarrow \frac{dV}{dx} = -3x^2 + A$$

$$= V = -x^3 + Ax + B$$

$V(1) = -50V \Rightarrow -50 = -1 + A + B \Rightarrow A + B = -49$
$V(4) = 50V \Rightarrow 50 = -64 + 4A + B \Rightarrow 4A + B = 114$

By solving above two equations

$$A + B = -49$$
$$4A + B = 114$$

$$A = 54.33, \; B = -103.33$$

$$V = -x^3 + 54.33x - 103.33$$

$$V(2) = -(2)^3 + 54.33(2) - 103.33 = -2.667 \text{ volts}$$

Resistance and Capacitance:

$$\bar{J} = \sigma \bar{E}$$

$$R = \frac{V}{I} = \frac{-\int \bar{E} \cdot d\bar{l}}{\int \bar{J} \cdot d\bar{s}}$$

$$= \frac{-\int \bar{E} \cdot d\bar{l}}{\sigma \int \bar{E} \cdot d\bar{s}}$$

$$C = \frac{Q}{V}$$

capacitance is defined as the ratio of the absolute value of charge to the absolute value of the voltage difference.

$$\bar{D} = \varepsilon \bar{E}$$

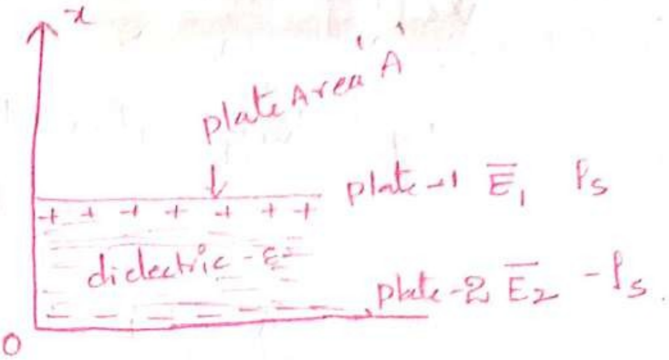
$$C = \frac{\int \bar{D} \cdot d\bar{s}}{-\int \bar{E} \cdot d\bar{l}} = \frac{\varepsilon \int \bar{E} \cdot d\bar{s}}{\int \bar{E} \cdot d\bar{l}}$$

$$\boxed{RC = \frac{-\int \bar{E} \cdot d\bar{l}}{\sigma \int \bar{E} \cdot d\bar{s}} \times \frac{\varepsilon \int \bar{E} \cdot d\bar{s}}{-\int \bar{E} \cdot d\bar{l}} = \frac{\varepsilon}{\sigma} = T_r}$$

T_r is the Relaxation time.

Capacitance of a parallel plate capacitor:

Consider a parallel plate capacitor as shown in figure. Each of the plates has an area 'A' and they are separated by distance 'd'.



We assume that plates 1 and 2, respectively carry charges +Q and −Q uniformly distributed on them. If the space between the plates is filled with a homogeneous dielectric with permittivity 'ε'

$$\bar{E}_1 = -\frac{\rho_s}{2\epsilon}\bar{a}_x$$

$$\bar{E}_2 = -\frac{\rho_s}{2\epsilon}\bar{a}_x \qquad \bar{E} = \bar{E}_1 + \bar{E}_2 = -\frac{\rho_s}{2\epsilon}\bar{a}_x - \frac{\rho_s}{2\epsilon}\bar{a}_x$$

$$\bar{E} = -\frac{\rho_s}{\epsilon}\bar{a}_x \qquad \boxed{\rho_s = \frac{Q}{A}}$$

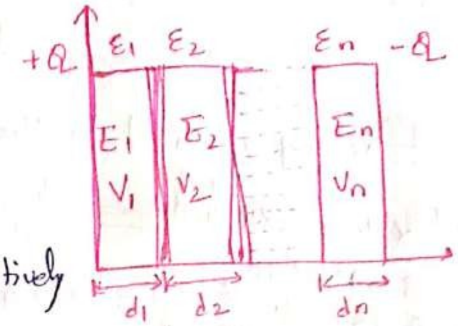
$$V = -\int_0^d \bar{E}\cdot d\bar{L} = -\int_0^d \left(-\frac{\rho_s}{\epsilon}\bar{a}_x\right)\cdot(dx\,\bar{a}_x) = \int_0^d \frac{\rho_s}{\epsilon}\,dx$$

$$V = \frac{\rho_s}{\epsilon}[x]_0^d = \frac{\rho_s d}{\epsilon} = \frac{Qd}{A\epsilon}$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\epsilon}} = \frac{A\epsilon}{d}$$

$$\boxed{C = \frac{A\epsilon}{d}}$$

Capacitance of parallel capacitor of n dielectric slabs



Proof: let $\varepsilon_1, \varepsilon_2, \varepsilon_3 \cdots \varepsilon_n$ be the permittivity of dielectric materials between the plates $d_1, d_2 \cdots d_n$ respectively be their thickness.

The potential difference across the capacitor is

$$V = V_1 + V_2 + \cdots V_n$$

$$\boxed{E = \frac{V}{d}}$$

$$V_1 = E_1 d_1$$
$$V_2 = E_2 d_2$$
$$V_n = E_n d_n$$

$$\boxed{D = \frac{Q}{A}} \quad \boxed{E = \frac{D}{\epsilon}}$$

$$V = E_1 d_1 + E_2 d_2 + \cdots + E_n d_n$$

$$V = \frac{D}{\epsilon_1} d_1 + \frac{D}{\epsilon_2} d_2 + \cdots + \frac{D}{\epsilon_n} d_n$$

$$V = \frac{Q}{A}\left[\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} + \cdots + \frac{d_n}{\epsilon_n}\right]$$

$$\boxed{C = \frac{Q}{V} = \frac{A}{\left[\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} + \cdots \frac{d_n}{\epsilon_n}\right]} = \frac{A}{\sum_{i=1}^{n}\left(\frac{d_i}{\epsilon_i}\right)} \text{ Farads}}$$

Problem: A parallel plate capacitor has conducting plates of area equal to $0.04\,m^2$. The plates are separated by a dielectric material whose $\epsilon_r = 2$ with plate separation of 1cm. Find a) its capacitance value b) The charge on the plates when a potential difference of 100 V is applied c) The energy stored.

Solution: a) $C = \dfrac{A\epsilon}{d}$

$$\varepsilon = \varepsilon_0 \varepsilon_r = 2 \times 8.856 \times 10^{-12}\ F/m$$
$$d = 1cm = 1 \times 10^{-2} m = 0.01 m$$
$$A = 0.04 m^2$$

$$C = \frac{0.04 \times 2 \times 8.856 \times 10^{-12}}{0.01} = 70.832 \times 10^{-12}$$

$$C = 70.832\,pF$$

b) $C = \dfrac{Q}{V} \Rightarrow Q = CV$

$= 70.832 \times 10^{-12} \times 100 = 7.0832 \times 10^{-7}$

$= 7.0832\ nC$

$= \cancel{708.32 \times 10^{-12}} = \cancel{708.32\ PC} = \cancel{7.0832 \times}$

c) Energy stored in the capacitor $W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV$

$= \frac{1}{2} \times 7.0832 \times 10^{-7} \times 100$ $\cancel{= \frac{1}{2}CV^2 = \frac{1}{2} \times 708.32 \times 10^{-9} \times (100)^2}$

$= 3.5416 \times 10^{-7}$ $= 0.354 \times 10^{-6}$

$= 0.35416 \times 10^{-6}$ $= \cancel{35.416 \times 10^{-8}} = \cancel{35.416\ nJoules} = \cancel{0.354\ \mu J}$oules.

$= 0.35416\ \mu J$

problem: Find the capacitance of a parallel plate capacitor when Area $1 m^2$ and the distance between the plates is 1mm. The voltage gradient is $10^5 V/m$ and the charge density on the plate is $2 \mu c/m^2$.

solution:

Given $A = 1 m^2$ $E = 10^5 V/m$

$d = 1mm = 10^{-3} m$

$\rho_s = 2 \mu c/m^2 = 2 \times 10^{-6} c/m^2$

The charge on the plates $Q = \rho_s A = 2 \times 10^{-6} \times 1 = 2 \times 10^{-6} C$

The potential $V = Ed = 10^5 \times 1 \times 10^{-3} = 100V$

$$C = \frac{Q}{V} = \frac{2 \times 10^{-6}}{100} = 2 \times 10^{-8} = 20 \times 10^{-9} = 20nF = 0.02\mu F$$

problem: Determine the capacitance of each of the parallel plate capacitor and Total capacitance in this capacitor with the following particulars: $A = 30 cm^2$, $d_1 = 2.5mm$, $\varepsilon_{r_1} = 4$, $d_2 = 2.5mm$, $\varepsilon_{r_2} = 6$

$\varepsilon_1 = \varepsilon_0 \varepsilon_{r_1}$

$\varepsilon_2 = \varepsilon_0 \varepsilon_{r_2}$

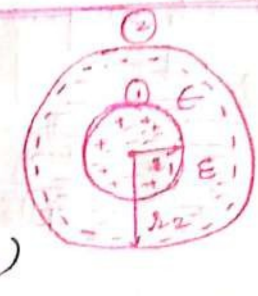
solution:

$$C = \frac{A}{\sum_{i=1}^{2} \frac{d_i}{\varepsilon_i}} = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}} = \frac{30 \times 10^{-4}}{\frac{2.5 \times 10^{-3}}{8.856 \times 10^{-12} \times 4} + \frac{2.5 \times 10^{-3}}{8.856 \times 10^{-12} \times 6}}$$

$$= \frac{30 \times 10^{-4} \times 8.856 \times 10^{-12}}{\frac{2.5 \times 10^{-3}}{4} + \frac{2.5 \times 10^{-3}}{6}} = \frac{2.6568 \times 10^{-14}}{1.0416 \times 10^{-3}} = 2.55 \times 10^{-11}$$

$= 25.5 \times 10^{-12} = 25.5\ PF$

Capacitance between two Concentric spheres:



Consider inner sphere of radius 'r_1' and outer sphere of radius 'r_2' ($r_2 > r_1$) seperated by a dielectric medium with permittivity 'ϵ' as shown in figure. Assume charges $+Q$ and $-Q$ on the inner and outer spheres respectively. $r_1 \le r \le r_2$

$$\bar{E} = \frac{Q}{4\pi\epsilon r^2}\bar{a}_r \qquad r_1 \le r \le r_2$$

potential difference between the two spheres is given by $V = -\int^{r_1} \bar{E}\cdot d\bar{l} \qquad d\bar{l} = dr\,\bar{a}_r$

$$= -\int_{r_2}^{r_1} \frac{Q}{4\pi\epsilon r^2}\bar{a}_r \cdot dr\,\bar{a}_r = \frac{Q}{4\pi\epsilon}\int_{r_2}^{r_1} -\frac{1}{r^2}dr$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{r}\right]_{r_2}^{r_1} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{Q[r_2 - r_1]}{4\pi\epsilon r_1 r_2}$$

$$\boxed{C = \frac{Q}{V} = \frac{4\pi\epsilon r_1 r_2}{(r_2 - r_1)} \text{ Farads.}}$$

Note: If $r_2 \to \infty$, the structure will result in an isolated sphere

The Capacitance of an isolated sphere of radius $r_1 = r$ is $\boxed{r_1 = r}$

$$\boxed{C = 4\pi\epsilon r \text{ Farads}}$$

problem: obtain the capacitance of an isolated sphere of radius 1cm.

solution: $C = 4\pi\epsilon r = 4\pi\epsilon_0 r = \frac{1\times10^{-2}}{9\times10^{9}} = 1.11\times10^{-12} = 1.11\,pF$

Problem: A spherical capacitor with $r_1 = 1.5$ cm $r_2 = 4$ cm has an inhomogeneous dielectric of $\epsilon = 10\epsilon_0/r$. Calculate the capacitance of the capacitor.

Solution $\bar{E} = \frac{Q}{4\pi\epsilon r^2}\bar{a_r}$

$r_1 = 1.5$ cm
$r_2 = 4$ cm
$\epsilon = 10\epsilon_0/r$

$d\bar{l} = dr\bar{a_r}$

$$V = -\int_{r_i}^{r_f} \bar{E}\cdot d\bar{l} = -\int_{r_i}^{r_f} \frac{Q}{4\pi\epsilon r^2}\bar{a_r}\cdot dr\bar{a_r} = -\int_{r_i}^{r_f} \frac{Q}{4\pi\epsilon r^2} dr$$

$$= -\int_{r_i}^{r_f} \frac{Q}{4\pi\frac{10\epsilon_0}{r} r^2} dr$$

$$= \frac{-Q}{40\pi\epsilon_0}\int_{r_i}^{r_f} \frac{1}{r} dr = \frac{-Q}{40\pi\epsilon_0}\left[\ln r\right]_{r_i}^{r_f}$$

$$V = \frac{-Q}{40\pi\epsilon_0}\left[\ln r_f/r_i\right] =$$

$$C = \frac{-40\pi\epsilon_0}{\ln\left(\frac{r_f}{r_i}\right)} = \frac{-40\pi\times 8.856\times 10^{-12}}{\ln\left(\frac{1.5}{4}\right)} = 1.135\times 10^{-9} = 1.135\text{nF}$$

Problem: The space between spherical conducting shells $r = 5$ cm and $r = 10$ cm is filled with a dielectric material for which $\epsilon = 2.25\epsilon_0$. The two shells are maintained at a potential difference of 80 V. a) Find the capacitance of the system b) calculate the charge density on shell $r = 5$ cm and $r = 10$ cm

$r_1 = 5\times 10^{-2}$ m
$r_2 = 10\times 10^{-2}$ m
$\epsilon = 2.25\epsilon_0$

Solution: a) $C = \frac{4\pi\epsilon r_1 r_2}{r_2 - r_1} = 4\pi\times 2.25\times 8.856\times 10^{-12}$

$$= \frac{4\pi\times 2.25\times 8.856\times 10^{-12}\times 5\times 10^{-2}\times 10\times 10^{-2}}{10\times 10^{-2} - 5\times 10^{-2}} = 25.02\times 10^{-12} = 25.02\text{pF}$$

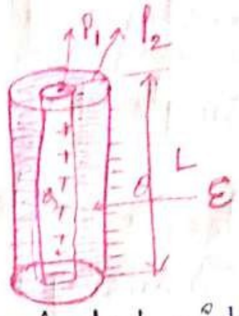
b) $Q = CV = 25.02\times 80\times 10^{-12} = 2\times 10^{-9}$ C

ρ_s on shell $r = 5$ cm $= \frac{Q}{4\pi r^2} = \frac{2\times 10^{-9}}{4\pi\times(25\times 10^{-4})} = 63.66\times 10^{-9}$ C/m^2

ρ_s on shell $r = 10$ cm $= \frac{Q}{4\pi r^2} = \frac{2\times 10^{-9}}{4\pi\times(10\times 10^{-2})^2} = 15.91\times 10^{-9}$ C/m^2

Capacitance of a coaxial cable:

Consider a coaxial cable of length 'L' of two coaxial conductors of inner radius ρ_1 and outer radius ρ_2 $(\rho_2 > \rho_1)$



Let the space between the conductors be filled with a homogeneous dielectric with permittivity ε. We assume the inner conductor carries $+Q$ and outer conductor carries $-Q$ uniformly distributed on them.

Inner conductor charge density ρ_L

Then the electric field in the radial direction is

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\bar{a}_\rho$$

The potential difference between the cylinders is

$$d\bar{L} = d\rho\,\bar{a}_\rho$$

$$V = -\int_{\rho_2}^{\rho_1} \bar{E}\cdot d\bar{L} = -\int_{\rho_2}^{\rho_1} \frac{\rho_L}{2\pi\varepsilon\rho}\bar{a}_\rho \cdot d\rho\,\bar{a}_\rho$$

$$V = -\int_{\rho_2}^{\rho_1} \frac{\rho_L}{2\pi\varepsilon\rho}\,d\rho = -\frac{\rho_L}{2\pi\varepsilon}\int_{\rho_2}^{\rho_1}\frac{1}{\rho}\,d\rho = \frac{-\rho_L}{2\pi\varepsilon}\left[\ln(\rho)\right]_{\rho_2}^{\rho_1}$$

$$V = \frac{-\rho_L}{2\pi\varepsilon}\ln(\rho_1/\rho_2) = \frac{\rho_L}{2\pi\varepsilon}\ln(\rho_2/\rho_1) \qquad \rho_L = Q/L$$

$$= \frac{Q}{2\pi\varepsilon L}\ln(\rho_2/\rho_1)$$

$$\boxed{C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln(\rho_2/\rho_1)} \text{ Farads}}$$

Problem: In an ink-jet printer the drops are charged by surrounding the jet of radius $20\mu m$ with a concentric cylinder of radius $600\mu m$ as required to generate a charge 50fC. Calculate the minimum voltage on the drop if the length of jet inside the cylinder is $100\mu m$. Take $\varepsilon = \varepsilon_0$

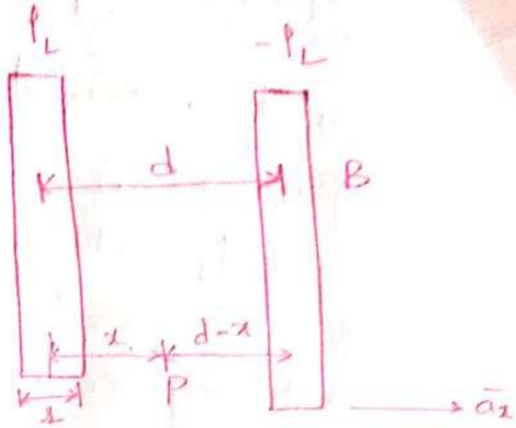
Solution:-

$L = 100\times10^{-6}\,m$, $\rho_2 = 600\times10^{-6}\,m$, $\rho_1 = 20\times10^{-6}\,\mu m$, $Q = 50\times10^{-15}$

$$C = \frac{2\pi\varepsilon L}{\ln(\rho_2/\rho_1)} = \frac{2\pi\times8.854\times10^{-12}\times100\times10^{-6}}{\ln\left(\frac{600\times10^{-6}}{20\times10^{-6}}\right)} = \underline{1.633\times10^{-15}}$$

$$V = \frac{Q}{C} = \cancel{\frac{1.633\times10^{-15}}{50\times10^{-15}}} = \frac{50\times10^{-15}}{1.633\times10^{-15}} = \underline{30.618 \text{ Volts}}$$

Capacitance of parallel wires:
(Single phase transmission line)



Consider a single phase transmission line comprising two parallel conducting wires, each of radius 'r' separated by a distance 'd' from their axis as shown in figure. Assume that the conductors are infinitely long. ρ_L and $-\rho_L$ are the charge densities on the conducting wires A and B respectively.

Let P be a point at a distance x from ρ_L along the x-axis.

The Electric field intensity at P due to ρ_L is $\bar{E_1} = \frac{\rho_L}{2\pi\varepsilon x}\bar{a_x}$

and that due to $-\rho_L$ with a distance $(d-x)$ is $\bar{E_2} = \frac{-\rho_L}{2\pi\varepsilon(d-x)}\bar{a_x}$

Total field at point P is $\bar{E} = \bar{E_1} + \bar{E_2}$

$$= \frac{\rho_L \bar{a_x}}{2\pi\varepsilon x} + \frac{\rho_L \bar{a_x}}{2\pi\varepsilon(d-x)} \Rightarrow \frac{\rho_L}{2\pi\varepsilon}\left[\frac{1}{x} - \frac{1}{(d-x)}\right]\bar{a_x}$$

$$V = -\int_{(d-r)}^{r} \frac{\rho_L}{2\pi\varepsilon x}\bar{a_x}\cdot dx\,\bar{a_x} + \int_{(d-r)}^{r} \frac{\rho_L}{2\pi\varepsilon(d-x)}\bar{a_x}\cdot dx\,\bar{a_x}$$

$$= -\int_{(d-r)}^{r} \frac{\rho_L}{2\pi\varepsilon x}dx + \int_{(d-r)}^{r} \frac{\rho_L}{2\pi\varepsilon(d-x)}dx \Rightarrow \frac{+\rho_L}{2\pi\varepsilon}\left[\int_{(d-r)}^{r}\left(-\frac{1}{x}\right)dx + \int_{(d-r)}^{r}\frac{1}{(d-x)}dx\right]$$

$$= \frac{\rho_L}{2\pi\varepsilon}\left[-[\ln(x)]_{(d-r)}^{r} + [\ln(d-x)]_{d-r}^{r}\right] = \frac{+\rho_L}{2\pi\varepsilon}\left[-\ln\left(\frac{r}{(d-r)}\right) + \ln\left(\frac{d-r}{r}\right)\right]$$

$$V = \frac{\rho_L}{2\pi\varepsilon}\left[\ln\left(\frac{d-r}{r}\right) + \ln\left(\frac{d-r}{r}\right)\right] = \frac{\rho_L}{\pi\varepsilon}\ln\left(\frac{d-r}{r}\right)$$

In all practical cases $d >> r$, As a result $\frac{d-r}{r} \simeq \frac{d}{r}$

$$V = \frac{Q}{\pi\varepsilon L}\ln\left(\frac{d-r}{r}\right) = \frac{Q}{\pi\varepsilon L}\ln(d/r) \qquad \boxed{\rho_L = \frac{Q}{L}}$$

$$\boxed{C = \frac{Q}{V} = \frac{\pi\varepsilon L}{\ln(d/r)}\ \text{Farad}}$$

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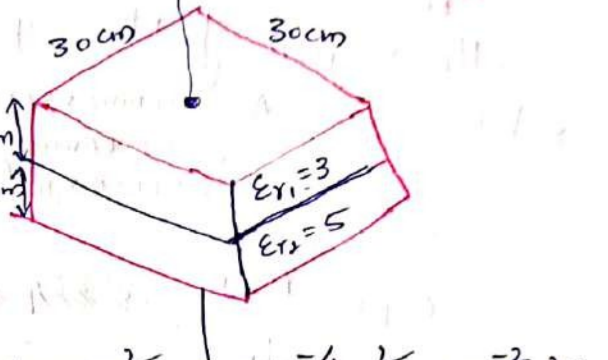
Problem: A parallel plate capacitor consisti of two plates, each 30mm x 30mm, spaced 2mm apart and 2 dielectrics, each 1mm thick, having relative perimittivites of 5 and 3 respectively. If the potential difference between the plates is 5000V, Calculate the voltage gradient in each dielectric



Solution:

Given the area of a capacitor

$$A = 30cm \times 30cm = 900cm^2 = 900\times10^{-4} m^2 = 9\times10^{-2} m^2$$

$d_1 = d_2 = 1mm = 1\times10^{-3} m$

$\varepsilon_{r_1} = 3,$

$\varepsilon_{r_2} = 5$

$V = 5000\ V$

The charge Q Remains same as Capacitors in Series

$$Q = VC_{eq} = V_1C_1 = V_2C_2$$

$$C_1 = \frac{A\epsilon_1}{d_1} = \frac{A\varepsilon_0\varepsilon_{r_1}}{d_1} = \frac{3\epsilon_0\times9\times10^{-2}}{10^{-3}} = 270\epsilon_0$$

$$C_2 = \frac{A\epsilon_2}{d_2} = \frac{A\varepsilon_0\varepsilon_{r_2}}{d_2} = \frac{5\epsilon_0\times9\times10^{-2}}{10^{-3}} = 450\epsilon_0$$

Total capacitance $C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{270\varepsilon_0\times450\epsilon_0}{270\varepsilon_0+450\epsilon_0} = 168.75\epsilon_0$

$$= 1.5\times10^{-9}\ \text{Farads}$$

If the V_1, V_2 are the potentials across the dielectrics

then $V_1 = \frac{VC_2}{C_1+C_2} = \frac{500\times450\times\epsilon_0}{(270\epsilon_0+450\epsilon_0)} = 3125$ volts

$V_2 = \frac{VC_1}{C_1+C_2} = \frac{5000\times270\times\epsilon_0}{(270\epsilon_0+450\epsilon_0)} = 1875$ volts

The voltage gradient [Electric field] across the first dielectric is

$E_1 = \frac{V_1}{d} = \frac{3125}{10^{-3}} = 3125\ kV/m$

Second dielectric is $E_2 = \frac{V_2}{d} = \frac{1875}{10^{-3}} = 1875\ kV/m$

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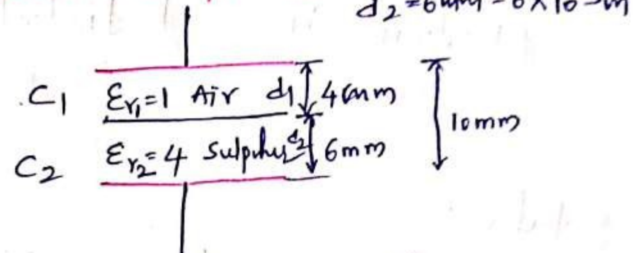
Problem: A parallel plate capacitance has 500mm side plates of square shape separated by 10mm distance. A sulfur slab of 6mm thickness with $\epsilon_r = 4$ is kept on the lower plate. Find the capacitance of the setup. If a voltage of 100V is applied across the capacitor, calculate the voltage at both the regions of the capacitor between the plates

$d_1 = 4mm = 4\times10^{-3}m$
$d_2 = 6mm = 6\times10^{-3}m$

Solution:

$A = 500mm \times 500mm$
$= 250000 mm^2$
$= 250000\times10^{-6} m^2$



$$C_1 = \frac{A\epsilon_1}{d_1} = 8.884\ \frac{A\epsilon_{r1}\epsilon_0}{d_1} = \frac{250000\times10^{-6}\times1\times8.854\times10^{-12}}{4\times10^{-3}}$$

$$= 0.5534\times10^{-9} \doteq 0.5534nF$$

$$C_2 = \frac{A\epsilon_2}{d_2} = \frac{250000\times10^{-6}\times4\times8.854\times10^{-12}}{6\times10^{-3}} = 1.4756\times10^{-9}$$
$$= 1.4756nF$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{0.553\times10^{-9}\times1.4756\times10^{-9}}{(0.5534\times10^{-9}+1.4756\times10^{-9})} = 0.4024\times10^{-9}F$$

$$Q = VC_{eq} = V_1C_1 = V_2C_2 \qquad \boxed{V_1+V_2 = V}$$

$$V_1 = \frac{VC_2}{C_1+C_2} = \frac{100\times1.4756\times10^{-9}}{(0.5534\times10^{-9}+1.4756\times10^{-9})} = 72.73 \text{ volts}$$

$$V_2 = \frac{VC_1}{C_1+C_2} = \frac{100\times0.5534\times10^{-9}}{(0.5534\times10^{-9}+1.4756\times10^{-9})} = 27.27 \text{ volts}$$

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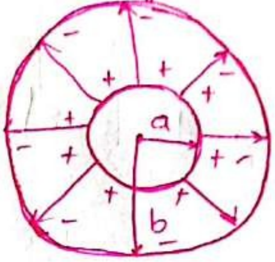
Conducting spherical shells with radii $a = 10\,cm$ and $b = 30\,cm$ are maintained at a potential difference of $100V$ such that $V(r=b) = 0$ and $V(r=a) = 100V$. Determine V and E in the region between the shells. If $\varepsilon_r = 2.5$ in the region, determine the total charge induced on the shells and the capacitance of the capacitor.



Solution: laplace's equation
(in spherical coordinate system) (r, θ, ϕ)

$$\nabla^2 V = 0$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

In this only 'r' is considered

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$\frac{d}{dr}\left[r^2 \frac{dV}{dr}\right] = 0$$

$$r^2 \frac{dV}{dr} = A \text{ (Constant)}$$

$$\frac{dV}{dr} = \frac{A}{r^2} \Rightarrow V = \frac{-A}{r} + B \text{ (Constant)}$$

when $r = b$, $V = 0 \Rightarrow 0 = \frac{-A}{b} + B$

$$\Rightarrow B = A/b$$

$$V = \frac{-A}{r} + \frac{A}{b} = A\left[\frac{1}{b} - \frac{1}{r}\right]$$

Also when $r = a$, $V = V_0 \Rightarrow V_0 = A\left[\frac{1}{b} - \frac{1}{a}\right]$

$$A = \frac{V_0}{\left[\frac{1}{b} - \frac{1}{a}\right]}$$

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$$\boxed{V = \frac{V_0\left[\frac{1}{r} - \frac{1}{b}\right]}{\left[\frac{1}{a} - \frac{1}{b}\right]}}$$

$d\bar{s} = r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r$

$$\boxed{\bar{E} = -\nabla V = -\frac{dv}{dr}\bar{a}_r = \frac{V_0}{r^2\left[\frac{1}{a} - \frac{1}{b}\right]}\bar{a}_r}$$

$$Q = \int \bar{D}\cdot d\bar{s} = \varepsilon \int \bar{E}\cdot ds = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{\varepsilon_0 \varepsilon_r V_0}{\cancel{r^2}\left[\frac{1}{a} - \frac{1}{b}\right]} \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$= \frac{\varepsilon_0 \varepsilon_r V_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}\left[-\cos\theta\right]_0^{\pi}\left[\phi\right]_0^{2\pi} = \frac{4\pi\varepsilon_0\varepsilon_r V_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\varepsilon_0\varepsilon_r}{\left[\frac{1}{a} - \frac{1}{b}\right]}}$$

substitute $a = 30\times10^{-2} = 0.3m$

$b = 10\times10^{-2} = 0.1m$

$V_0 = 100\,v$

$$V = \frac{100\left[\frac{1}{r} - \frac{1}{0.3}\right]}{\left[\frac{1}{0.1} - \frac{1}{0.3}\right]} = \frac{100\left[\frac{1}{r} - \frac{10}{3}\right]}{\left[10 - \frac{10}{3}\right]} = 15\left[\frac{1}{r} - \frac{10}{3}\right] \text{ volts}$$

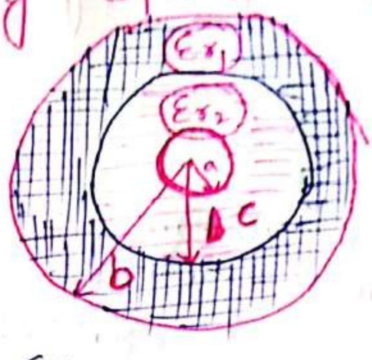
$$\bar{E} = \frac{100}{r^2\left[\frac{1}{0.1} - \frac{1}{0.3}\right]}\bar{a}_r = \frac{100}{r^2\left[10 - \frac{10}{3}\right]}\bar{a}_r = \frac{15}{r^2}\bar{a}_r$$

$$Q = \frac{4\pi\times2.5\times8.854\times10^{-12}\times100}{\left[\frac{1}{0.1} - \frac{1}{0.3}\right]} = 4.167\times10^{-9} \text{ Coulomb}$$

positive charge is induced on the inner shell
negative charge is induced on the outer shell

$$C = \frac{Q}{V} = \frac{4.167\times10^{-9}}{100} = 41.67\times10^{-12} = 41.67\,PF$$

Problem: Determine the capacitance of spherical capacitor as shown in the figure



Solution

$\varepsilon_{r_1} = 2.5$

$\varepsilon_{r_2} = 3.5$

$a = 1\,mm = 1\times10^{-3}\,m$

$b = 3\,mm = 3\times10^{-3}\,m$

$c = 2\,mm = 2\times10^{-3}\,m$

$\varepsilon_1 = \varepsilon_0 \varepsilon_{r_1}$

$\varepsilon_2 = \varepsilon_0 \varepsilon_{r_2}$

Capacitance of sphere with inner radius 'a' and outer radius 'c' with $\varepsilon_{r_2} = 3.5$

$$C_1 = \frac{4\pi\varepsilon_1 ac}{(c-a)} = \frac{4\pi\times3.5\times8.854\times10^{-12}\times10^{-3}\times2\times10^{-3}}{(2\times10^{-3}-1\times10^{-3})}$$

$$= 0.784\times10^{-12}\ \text{Farad} = 0.784\ \text{PF}$$

Capacitance of sphere with innerradius 'c' and outerradius 'b' with $\varepsilon_{r_1} = 2.5$

$$C_2 = \frac{4\pi\varepsilon_2 bc}{(b-c)} = \frac{4\pi\times2.5\times8.854\times10^{-12}\times3\times10^{-3}\times2\times10^{-3}}{(3\times10^{-3}-2\times10^{-3})}$$

$$= 1.668\times10^{-12}\ \text{Farad} = 1.668\ \text{PF}$$

C_1 and C_2 are in series so that the equivalent Capacitance is 'c' $= C_{eq} = \dfrac{C_1C_2}{C_1+C_2}$

$$= \frac{1.668\times10^{-12}\times0.784\times10^{-12}}{(1.668\times10^{-12}+0.784\times10^{-12})}$$

$$= 0.533\times10^{-12} = 0.533\ \text{PF}$$

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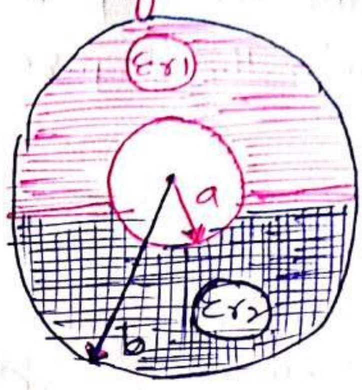
Determine the capacitance of the following spherical capacitor:



Solution:

$\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 3.5$

$a = 1mm = 1\times10^{-3}m$

$b = 3mm = 3\times10^{-3}m$

$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$

$\varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$

The capacitance of the sphere with $\varepsilon_{r1} = 2.5$

$$C_1 = \frac{1}{2} \times \frac{4\pi\varepsilon_1 ab}{b-a} = \frac{1}{2} \times \frac{4\times3.14\times2.5\times8.854\times10^{-12}\times1\times10^{-3}\times3\times10^{-3}}{(3\times10^{-3}-2\times10^{-3})}$$

$$= 0.2085\times10^{-12} = \underline{0.2085\,pF}$$

The capacitance of the sphere with $\varepsilon_{r2} = 3.5$

$$C_2 = \frac{1}{2} \times \frac{4\pi\varepsilon_2 ab}{(b-a)} = \frac{1}{2} \times \frac{4\times3.14\times3\times8.854\times10^{-12}\times10^{-3}\times3\times10^{-3}}{(3\times10^{-3}-2\times10^{-3})}$$

$$= 0.2919\times10^{-12} = \underline{0.2919\,pF}$$

C_1, C_2 are parallel

Total capacitance 'C' $= C_{eq} = C_1 + C_2$

$$= 0.2085\times10^{-12} + 0.2919\times10^{-12}$$

$$= 0.5004\times10^{-12} = \underline{0.5\,pF}$$

Problem: A cylindrical capacitor has radii $a = 1\,cm$ and $b = 2.5\,cm$. If the space between the plates is filled with an inhomogeneous dielectric with $\varepsilon_r = \left(\frac{100+\rho}{\rho}\right)$, where ρ is in centimeters, find the capacitance per meter of the capacitor.

Solution:

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\bar{a}_\rho \qquad \boxed{\rho_L = \frac{Q}{L}} \qquad \boxed{d\bar{L} = d\rho\,\bar{a}_\rho} \qquad \boxed{\varepsilon = \varepsilon_0\varepsilon_r}$$

$$V = -\int_b^a \bar{E}\cdot d\bar{L} = -\int_b^a \frac{Q}{2\pi\varepsilon\rho L}\bar{a}_\rho \cdot d\rho\,\bar{a}_\rho$$

$$= -\int_b^a \frac{Q\,d\rho}{2\pi\varepsilon_0\left(\frac{100+\rho}{\rho}\right)\cdot\rho\cdot L} = \frac{-Q}{2\pi\varepsilon_0 L}\int_b^a \frac{d\rho}{(100+\rho)}$$

$$= \frac{-Q}{2\pi\varepsilon_0 L}\Big|\ln(100+\rho)\Big|_b^a = \frac{-Q}{2\pi\varepsilon_0 L}\ln\left[\frac{(100+a)}{(100+b)}\right]$$

$$V = \frac{Q}{2\pi\varepsilon_0 L}\ln\left[\frac{100+b}{100+a}\right]$$

ρ is in centimeters, so that substitute a, b in centimeters

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\left[\frac{100+b}{100+a}\right]}$$

$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left[\frac{100+b}{100+a}\right]} = \frac{2\pi\times 8.854\times10^{-12}}{\ln\left[\frac{10+2.5}{10+1}\right]} = 434.6\times10^{-12}\ F/m$$

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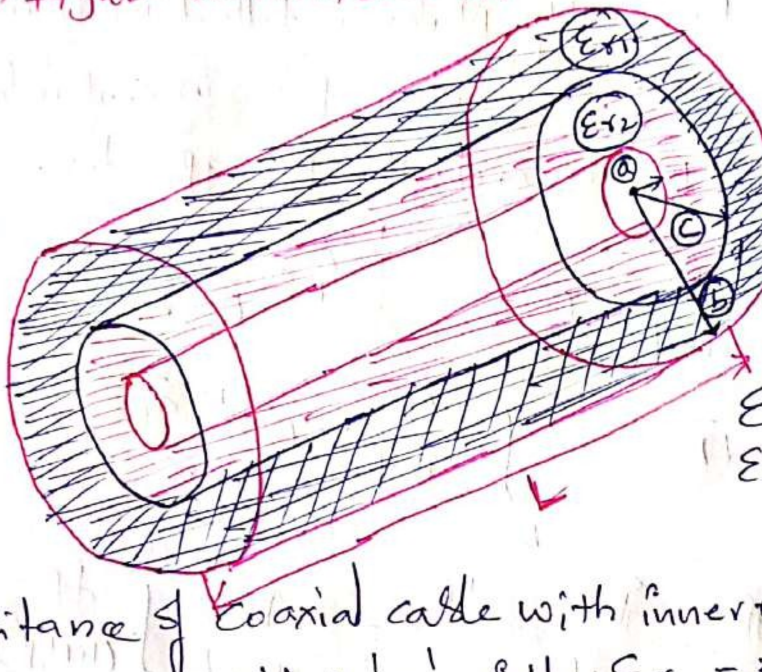
problem:
Determine the Capacitance of 10m length of the Cylindrical capacitor as shown in figure $a = 1mm, b = 3mm, c = 2mm$ $\varepsilon_{r1} = 2.5$ and $\varepsilon_{r2} = 3.5$

solution:



$L = 10m$
$\varepsilon_{r1} = 2.5$
$\varepsilon_{r2} = 3.5$
$a = 1mm = 1\times10^{-3}m$
$b = 3mm = 3\times10^{-3}m$
$c = 2mm = 2\times10^{-3}mm$
$\varepsilon_0 = 8.854\times10^{-12} F/m$
$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$
$\varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$

capacitance of coaxial cable with inner ratius 'a' and outer ratius 'c' with $\varepsilon_{r2} = 3.5$

$$C_1 = \frac{2\pi\varepsilon_1 L}{\ln\left(\frac{c}{a}\right)} = \frac{2\pi\times8.854\times10^{-12}\times2.5\times10}{\ln\left(\frac{2\times10^{-3}}{1\times10^{-3}}\right)} = 2.0054\times10^{-9} F$$
$$= 2nF$$

Capacitance of Coaxial Cable with inner ratius 'c' and outer ratius 'b' with $\varepsilon_{r1} = 2.5$

$$C_2 = \frac{2\pi\varepsilon_2 L}{\ln\left(\frac{b}{c}\right)} = \frac{2\pi\times8.854\times10^{-12}\times3.5\times10}{\ln\left(\frac{3\times10^{-3}}{2\times10^{-3}}\right)} = 4.8\times10^{-9} F$$
$$= 4.8 nF$$

C_1 and C_2 are in series, so that the equivalent Capacitance is 'C' $= C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

$$= \frac{4.8\times10^{-9}\times2\times10^{-9}}{4.8\times10^{-9}+2\times10^{-9}} = 1.41\times10^{-9}$$

$C_{eq} = 1.41 nF$

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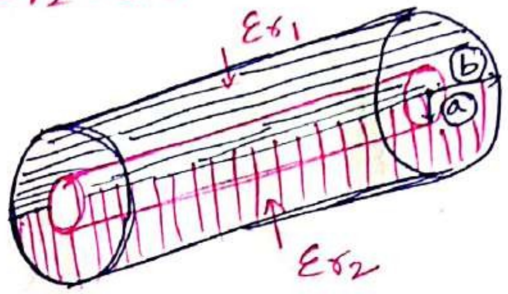
problem: Determine the capacitance of 10m length of the cylindrical capacitors shown figure $a = 1\,mm$, $b = 3\,mm$, $\varepsilon_{r1} = 2.5$ and $\varepsilon_{r2} = 3.5$



$\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 3.5$

$a = 1mm = 1\times10^{-3}m$

$b = 3mm = 3\times10^{-3}m$

$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$

$\varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$

The Capacitance of the coaxial cable with $\varepsilon_{r1} = 2.5$

$$C_1 = \frac{1}{2}\times\frac{2\pi\varepsilon_1 L}{\ln(b/a)} = \frac{1}{2}\times\frac{2\pi\times8.854\times10^{-12}\times2.5\times10}{\ln\left(\frac{3\times10^{-3}}{1\times10^{-3}}\right)} = \frac{1.265\times10^{-9}}{2}$$

$$= \cancel{0.645\times10^{-9}F} = \cancel{0.632\,nF} \quad = 0.6325\times10^{-9} = 0.6325\,nF$$

The capacitance of the coaxial cable with $\varepsilon_{r2} = 3.5$

$$C_2 = \frac{1}{2}\times\frac{2\pi\varepsilon_2 L}{\ln\left(\frac{b}{a}\right)} = \frac{1}{2}\times\frac{2\pi\times8.854\times10^{-12}\times3.5\times10}{\ln\left(\frac{3\times10^{-3}}{1\times10^{-3}}\right)} = 0.8857\times10^{-9} = 0.8857\,nF$$

C_1, C_2 are parallel

Total capacitance $'C' = C_{eq} = C_1 + C_2$

$$= 0.6325\times10^{-9} + 0.8857\times10^{-9}$$

$$= 1.518\times10^{-9} = 1.518\,nF$$